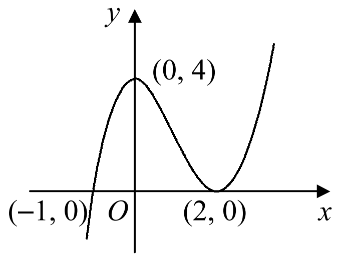
1 **a**



b $f(x) = (x + 1)(x^2 - 4x + 4)$

$= x^3 - 4x^2 + 4x + x^2 - 4x + 4$

$= x^3 - 3x^2 + 4$

$f'(x) = 3x^2 - 6x$

c $x = 1 \therefore y = 2 \times (-1)^2 = 2$

grad $= 3 - 6 = -3$

$\therefore y - 2 = -3(x - 1)$

$y - 2 = -3x + 3$

$y = 5 - 3x$

2 **a** $\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$

grad at $P = \frac{1}{4}$

$\therefore y - 1 = \frac{1}{4}(x - 4)$

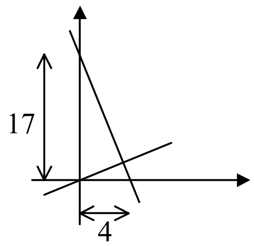
$y = \frac{1}{4}x$ which passes through (0, 0)

b grad of normal $= -4$

$\therefore y - 1 = -4(x - 4)$ [$y = 17 - 4x$]

at Q, $x = 0 \Rightarrow y = 17$

$\therefore$ area $= \frac{1}{2} \times 17 \times 4 = 34$



3 **a** $x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, 1$ $a < b$ $\therefore a = -2, b = 1$

b $\frac{dy}{dx} = 2x + 1$

grad at $A = -3$

$\therefore$ grad of normal $= \frac{1}{3}$

$\therefore y - 0 = \frac{1}{3}(x + 2)$

$3y = x + 2$

$x - 3y + 2 = 0$

c grad at $B = 3$

tangent at B: $y - 0 = 3(x - 1)$

$y = 3x - 3$

at C, $x - 3(3x - 3) + 2 = 0$

$x = \frac{11}{8}$

$\therefore C(\frac{11}{8}, \frac{9}{8})$

4 $y = \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{x^2 - 2x + 1}{2x^{\frac{3}{2}}}$

$= \frac{(x-1)^2}{2x^{\frac{3}{2}}}$ [$a = -1, b = 2$]

5 **a** $\frac{dy}{dx} = -24x^{-3}$

at A, $y = 3$, grad$= -3$

$\therefore y - 3 = -3(x - 2)$

$3x + y - 9 = 0$

b tangent:

$x = -1 \Rightarrow -3 + y - 9 = 0 \Rightarrow y = 12$

curve:

$x = -1 \Rightarrow y = \frac{12}{1} \Rightarrow y = 12$

$\therefore$ tangent intersects curve at (−1, 12)

6 **a** $\frac{dy}{dx} = 3 + 2kx - 3x^2$

at P, $3 - 2k - 3 = -6$

$k = 3$

b $y = 2 + 3x + 3x^2 - x^3$ $\therefore P(-1, 3)$

at Q, $3 + 6x - 3x^2 = -6$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1$ (at P) or 3 $\therefore Q(3, 11)$

$PQ = \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$

7 $= \frac{d}{dx}(x^2 + \frac{1}{2}x^{-1})$

$= 2x - \frac{1}{2}x^{-2}$

8 **a** $\frac{dy}{dx} = 4x - 7$

at A, $y = -5$, grad $= 1$

$\therefore\ y + 5 = 1(x - 2)$

$[\ y = x - 7\ ]$

b grad of normal at $B = 1$

$\therefore$ grad of curve at $B = -1$

$\therefore\ 4x - 7 = -1$

$x = \frac{3}{2},\ y = 2(\frac{9}{4}) - 7(\frac{3}{2}) + 1 = -5$

$\therefore\ B\,(\frac{3}{2}, -5)$

9 **a** $\frac{dy}{dx} = 2x + \frac{3}{2}x^{-\frac{1}{2}}$

b $\frac{d^2y}{dx^2} = 2 - \frac{3}{4}x^{-\frac{3}{2}}$

$\therefore\ 2x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6x$

$= 2x(2 - \frac{3}{4}x^{-\frac{3}{2}}) + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$

$= 4x - \frac{3}{2}x^{-\frac{1}{2}} + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$

$= 0$

10 **a** $\frac{dy}{dx} = -4x^{-2}$

grad at $M = -\frac{1}{4}$

$\therefore$ grad of normal $= 4$

$\therefore\ y - 3 = 4(x - 4)$ $[\ y = 4x - 13]$

b $4x - 13 = 2 + \frac{4}{x}$

$4x^2 - 15x - 4 = 0$

$(4x + 1)(x - 4) = 0$

$x = 4$ (at M) or $-\frac{1}{4}$

$\therefore\ N\,(-\frac{1}{4}, -14)$

11 **a** $\frac{dy}{dx} = 3x^2 - 6x - 8$

grad at $P = 1$

$\therefore\ y - 8 = 1(x + 1)$ $[\ y = x + 9]$

b at Q, $3x^2 - 6x - 8 = 1$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1$ at P $\therefore\ Q\,(3, -20)$

$\therefore\ y + 20 = 1(x - 3)$ $[\ y = x - 23]$

c grad normal $= -1$

$\therefore\ y - 8 = -(x + 1)$ $[\ y = 7 - x]$

d normal at P meets m when

$7 - x = x - 23$

$x = 15$ $\therefore\ (15, -8)$

dist between lines = dist P to $(15, -8)$

$= \sqrt{16^2 + 16^2} = \sqrt{16^2 \times 2} = 16\sqrt{2}$

12 **a** $y = kx^{\frac{1}{2}} - x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

at P, $\frac{1}{2}k(\frac{1}{\sqrt{2}}) - \frac{3}{2}(\sqrt{2}) = \sqrt{2}$

$k - 6 = 4$

$k = 10$

b $y = \sqrt{x}\,(10 - x)$

at P, $y = \sqrt{2}\,(10 - 2) = 8\sqrt{2}$

grad of normal $= -\frac{1}{\sqrt{2}}$

$\therefore\ y - 8\sqrt{2} = -\frac{1}{\sqrt{2}}(x - 2)$

$\sqrt{2}\,y - 16 = -x + 2$

$x + \sqrt{2}\,y = 18$ $[\ c = 18\ ]$