

1 a volume =  $2x^2h = 4000$

$$\therefore h = \frac{2000}{x^2}$$

b  $A = 2x^2 + 2(2xh) + 2(xh)$

$$= 2x^2 + 6xh$$

$$= 2x^2 + (6x \times \frac{2000}{x^2})$$

$$= 2x^2 + \frac{12000}{x}$$

c  $\frac{dA}{dx} = 4x - 12000x^{-2}$

SP:  $4x - 12000x^{-2} = 0$

$$x^3 = 3000$$

$$x = \sqrt[3]{3000} = 14.4 \text{ (3sf)}$$

d  $\min A = 1250 \text{ (3sf)}$

e  $\frac{d^2A}{dx^2} = 4 + 24000x^{-3}$

when  $x = \sqrt[3]{3000}$ ,  $\frac{d^2A}{dx^2} = 12$

$$\frac{d^2A}{dx^2} > 0 \therefore \text{minimum}$$

3 a S.A. =  $2x^2 + 4xl = k$

$$\therefore 4xl = k - 2x^2$$

$$l = \frac{k - 2x^2}{4x}$$

b  $V = x^2l$

$$= x^2 \times \frac{k - 2x^2}{4x}$$

$$= \frac{1}{4}kx - \frac{1}{2}x^3$$

$$\frac{dV}{dx} = \frac{1}{4}k - \frac{3}{2}x^2$$

SP:  $\frac{1}{4}k - \frac{3}{2}x^2 = 0$

$$x^2 = \frac{1}{6}k$$

$$x = \sqrt{\frac{k}{6}}$$

$$\frac{d^2V}{dx^2} = -3x$$

when  $x = \sqrt{\frac{k}{6}}$ ,  $\frac{d^2V}{dx^2} < 0 \therefore \text{maximum}$

$$l = \frac{k - \frac{1}{3}k}{4\sqrt{\frac{k}{6}}} = \frac{2}{3}k \times \frac{1}{4} \times \sqrt{\frac{6}{k}}$$

$$= \frac{k}{6} \times \sqrt{\frac{6}{k}} = \sqrt{\frac{k}{6}}$$

$\therefore$  maximum  $V$  when  $l = x \therefore$  prism is a cube

2 a S.A. =  $2\pi r^2 + 2\pi rh = 30\,000$

$$\therefore \pi rh = 15\,000 - \pi r^2$$

$$h = \frac{15000}{\pi r} - r$$

$$V = \pi r^2 h$$

$$= \pi r^2 \left( \frac{15000}{\pi r} - r \right)$$

$$= 15\,000r - \pi r^3$$

b  $\frac{dV}{dr} = 15\,000 - 3\pi r^2$

SP:  $15\,000 - 3\pi r^2 = 0$

$$r^2 = \frac{5000}{\pi}$$

$$r = \sqrt{\frac{5000}{\pi}} \quad [= 39.9 \text{ (3sf)}]$$

max volume =  $399\,000 \text{ cm}^3 \text{ (3sf)}$

$$\frac{d^2V}{dr^2} = -6\pi r$$

when  $r = \sqrt{\frac{5000}{\pi}}$ ,  $\frac{d^2V}{dr^2} = -752$

$$\frac{d^2V}{dr^2} < 0 \therefore \text{maximum}$$