

- 1 a  $f'(x) = 24 + 6x - 3x^2$   
 b  $24 + 6x - 3x^2 \geq 0$   
 $x^2 - 2x - 8 \leq 0$   
 $(x + 2)(x - 4) \leq 0$   
 $-2 \leq x \leq 4$
- 2 a  $(-2, 30) \Rightarrow 30 = -8 + 4a + 48 + b$   
 $\therefore 4a + b + 10 = 0$   
 b  $\frac{dy}{dx} = 3x^2 + 2ax - 24$   
 SP at P  $\therefore \frac{dy}{dx} = 0$   
 $\Rightarrow 12 - 4a - 24 = 0$   
 $a = -3, b = 2$   
 c  $3x^2 - 6x - 24 = 0$   
 $3(x + 2)(x - 4) = 0$   
 $x = -2$  (at P) or 4  
 other SP (4, -78)
- 3 a  $f'(x) = 2x - 16x^{-2}$   
 b SP:  $2x - 16x^{-2} = 0$   
 $x^3 = 8$   
 $x = 2$   
 $\therefore (2, 12)$   
 $f''(x) = 2 + 32x^{-3}$   
 $f''(2) = 6$   
 $f''(x) > 0 \therefore$  minimum
- 4 a area =  $(2 \times \frac{1}{2}r^2\theta) + \frac{1}{2}r^2(3\theta) = 25$   
 $\therefore \frac{5}{2}r^2\theta = 25, \theta = \frac{10}{r^2}$   
 b  $P = 2r + (2 \times r\theta) + r(3\theta) = 2r + 5r\theta$   
 $= 2r + 5r(\frac{10}{r^2}) = 2r + \frac{50}{r}$   
 c  $\frac{dP}{dr} = 2 - 50r^{-2}$   
 SP:  $2 - 50r^{-2} = 0$   
 $r^2 = 25$   
 $r = 5$   
 d min  $P = 20$   
 e  $\frac{d^2P}{dr^2} = 100r^{-3}$ , when  $r = 5, \frac{d^2P}{dr^2} = 0.8$   
 $\frac{d^2P}{dr^2} > 0 \therefore$  minimum
- 5 a  $2x - x^{\frac{3}{2}} = 0$   
 $x(2 - x^{\frac{1}{2}}) = 0$   
 $x = 0$  or  $x^{\frac{1}{2}} = 2 \Rightarrow x = 4$   
 $\therefore (0, 0)$  and  $(4, 0)$   
 b  $\frac{dy}{dx} = 2 - \frac{3}{2}x^{\frac{1}{2}}$   
 SP:  $2 - \frac{3}{2}x^{\frac{1}{2}} = 0$   
 $x^{\frac{1}{2}} = \frac{4}{3}$   
 $x = \frac{16}{9}$   
 $\frac{d^2y}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$ , when  $x = \frac{16}{9}, \frac{d^2y}{dx^2} = -\frac{9}{16}$   
 $\frac{d^2y}{dx^2} < 0 \therefore$  maximum
- 6 a  $\frac{dy}{dx} = 3x^2 - 3$   
 SP:  $3x^2 - 3 = 0$   
 $x^2 = 1$   
 $x = \pm 1$   
 $\therefore (-1, 3)$  and  $(1, -1)$   
 b  $PQ^2 = 2^2 + 4^2 = 20$   
 $\therefore PQ = \sqrt{20} = 2\sqrt{5}$
- c

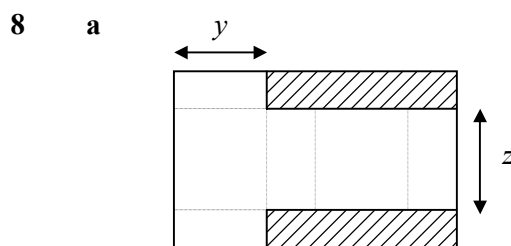
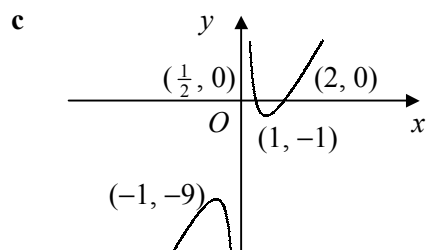
7 a  $2x - 5 + \frac{2}{x} = 0$

$$2x^2 - 5x + 2 = 0$$

$$(2x - 1)(x - 2) = 0$$

$$x = \frac{1}{2}, 2$$

b  $f'(x) = 2 - 2x^{-2}$   
 $\therefore 2 - 2x^{-2} = 0$   
 $x^2 = 1$   
 $x = \pm 1$



$$2x + z = 25$$

$$2x + 2y = 40$$

$\therefore$  length and width  $(25 - 2x)$  and  $(20 - x)$

b volume  $= x(25 - 2x)(20 - x)$   
 $= x(500 - 65x + 2x^2)$   
 $= 2x^3 - 65x^2 + 500x$

c  $\frac{dV}{dx} = 6x^2 - 130x + 500$   
 SP:  $6x^2 - 130x + 500 = 0$   
 $2(3x - 50)(x - 5) = 0$   
 $x = 5, \frac{50}{3}$

$$2x < 25 \quad \therefore x < 12.5$$

$$\therefore x = 5$$

d max volume  $= 1125 \text{ cm}^3$

$$\frac{d^2V}{dx^2} = 12x - 130$$

when  $x = 5$ ,  $\frac{d^2V}{dx^2} = -70$

$$\frac{d^2V}{dx^2} < 0 \quad \therefore \text{maximum}$$

9 a  $\frac{dy}{dx} = 9 + 6x - 3x^2$   
 SP:  $9 + 6x - 3x^2 = 0$   
 $-3(x + 1)(x - 3) = 0$   
 $x = -1, 3$

$$\therefore (-1, -3) \text{ and } (3, 29)$$

b  $\frac{d^2y}{dx^2} = 6 - 6x$

$(-1, -3)$ :  $\frac{d^2y}{dx^2} = 12 \quad \therefore$  minimum

$(3, 29)$ :  $\frac{d^2y}{dx^2} = -12 \quad \therefore$  maximum

c  $-3 < k < 29$

10 a  $f(-1) = 15$   
 $\therefore -4 + a + 12 + b = 15$   
 $a + b = 7 \quad (1)$

b  $f(2) = 42$   
 $\therefore 32 + 4a - 24 + b = 42$   
 $4a + b = 34 \quad (2)$

$$(2) - (1) \quad 3a = 27$$

$$\therefore a = 9, b = -2$$

c  $f(x) = 4x^3 + 9x^2 - 12x - 2$   
 $f'(x) = 12x^2 + 18x - 12$   
 SP:  $12x^2 + 18x - 12 = 0$   
 $2x^2 + 3x - 2 = 0$   
 $(2x - 1)(x + 2) = 0$   
 $x = -2, \frac{1}{2}$   
 $\therefore (-2, 26) \text{ and } (\frac{1}{2}, -\frac{21}{4})$