

1 a $x^2 + x + c$

b $y = x^2 + x + c$

$$(1, 5) \Rightarrow 5 = 1 + 1 + c$$

$$\therefore c = 3$$

$$y = x^2 + x + 3$$

2 a $y = \int (3 - 6x) \, dx$

$$y = 3x - 3x^2 + c$$

$$(2, 1) \Rightarrow 1 = 6 - 12 + c$$

$$\therefore c = 7$$

$$y = 3x - 3x^2 + 7$$

b $y = \int (3x^2 - x) \, dx$

$$y = x^3 - \frac{1}{2}x^2 + c$$

$$(4, 41) \Rightarrow 41 = 64 - 8 + c$$

$$\therefore c = -15$$

$$y = x^3 - \frac{1}{2}x^2 - 15$$

c $y = \int (x^2 + 4x + 1) \, dx$

$$y = \frac{1}{3}x^3 + 2x^2 + x + c$$

$$(-3, 4) \Rightarrow 4 = -9 + 18 - 3 + c$$

$$\therefore c = -2$$

$$y = \frac{1}{3}x^3 + 2x^2 + x - 2$$

d $y = \int (7 - 5x - x^3) \, dx$

$$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4 + c$$

$$(2, 0) \Rightarrow 0 = 14 - 10 - 4 + c$$

$$\therefore c = 0$$

$$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4$$

e $y = \int (8x - 2x^{-2}) \, dx$

$$y = 4x^2 + 2x^{-1} + c$$

$$\left(\frac{1}{2}, -1\right) \Rightarrow -1 = 1 + 4 + c$$

$$\therefore c = -6$$

$$y = 4x^2 + 2x^{-1} - 6$$

f $y = \int (3 - x^{\frac{1}{2}}) \, dx$

$$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + c$$

$$(4, 8) \Rightarrow 8 = 12 - \frac{16}{3} + c$$

$$\therefore c = \frac{4}{3}$$

$$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}$$

3 $f(x) = \int (3 + 2x - x^2) \, dx$

$$f(x) = 3x + x^2 - \frac{1}{3}x^3 + c$$

$$(3, 5) \Rightarrow 5 = 9 + 9 - 9 + c$$

$$\therefore c = -4$$

$$f(x) = 3x + x^2 - \frac{1}{3}x^3 - 4$$

4 $y = \int (10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}) \, dx$

$$y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$$

$$y = 0 \text{ when } x = 7$$

$$\therefore 7 = 0 + 0 + c$$

$$c = 7$$

$$\therefore y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + 7$$

$$\text{when } x = 4$$

$$y = 4(32) - 4(2) + 7$$

$$y = 127$$

5 a $f(x) = \int (2x^3 - x - 8) dx$
 $f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x + c$
 $(-1, 4) \Rightarrow 4 = \frac{1}{2} - \frac{1}{2} + 8 + c$
 $\therefore c = -4$
 $f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x - 4$

b at $x = 2$, $y = 8 - 2 - 16 - 4 = -14$
 $\text{grad} = 16 - 2 - 8 = 6$
 $\therefore y + 14 = 6(x - 2)$
 $[y = 6x - 26]$

7 a $y = \int (3x + 2x^{-2}) dx$
 $y = \frac{3}{2}x^2 - 2x^{-1} + c$

b $y = 8$ when $x = 2$
 $\therefore 8 = 6 - 1 + c$
 $c = 3$
 $\therefore y = \frac{3}{2}x^2 - 2x^{-1} + 3$
 when $x = \frac{1}{2}$
 $y = \frac{3}{8} - 4 + 3$
 $y = -\frac{5}{8}$

6 $f(x) = \int (3x^2 - 8x - 5) dx$
 $f(x) = x^3 - 4x^2 - 5x + c$
 $(0, 0) \Rightarrow 0 = 0 + c$
 $\therefore c = 0$
 $f(x) = x^3 - 4x^2 - 5x$
 $= x(x^2 - 4x - 5)$
 $= x(x + 1)(x - 5)$
 crosses x -axis when $f(x) = 0$
 $\therefore (-1, 0)$ and $(5, 0)$

8 a $y = \int (3x^2 + kx) dx$
 $y = x^3 + \frac{1}{2}kx^2 + c$
 $(1, 6) \Rightarrow 6 = 1 + \frac{1}{2}k + c$
 $5 = \frac{1}{2}k + c \quad (1)$
 $(2, 1) \Rightarrow 1 = 8 + 2k + c$
 $-7 = 2k + c \quad (2)$
 $(2) - (1) \quad -12 = \frac{3}{2}k$
 $k = -8$

b sub. $-7 = -16 + c$
 $c = 9$
 $\therefore y = x^3 - 4x^2 + 9$