

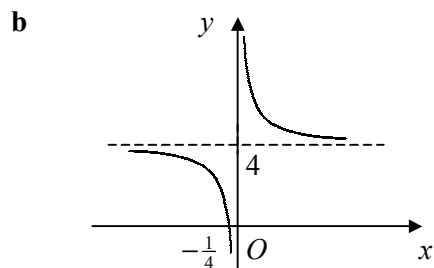
$$1 \quad \text{a} \quad = [-2x^{-1}]_1^4 \\ = -\frac{1}{2} - (-2) \\ = \frac{3}{2}$$

$$\text{b} \quad = \int_0^2 (x^2 - 6x + 9) \, dx \\ = [\frac{1}{3}x^3 - 3x^2 + 9x]_0^2 \\ = (\frac{8}{3} - 12 + 18) - 0 \\ = 8\frac{2}{3}$$

$$3 \quad \text{a} \quad = 3\sqrt{2} - \frac{1}{\sqrt{2}} \\ = 3\sqrt{2} - \frac{1}{2}\sqrt{2} \\ = \frac{5}{2}\sqrt{2}$$

$$\text{b} \quad \int_3^4 (3x^{\frac{1}{2}} - x^{-\frac{1}{2}}) \, dx \\ = [2x^{\frac{3}{2}} - 2x^{\frac{1}{2}}]_3^4 \\ = [16 - 4] - [(2 \times 3\sqrt{3}) - 2\sqrt{3}] \\ = 12 - 4\sqrt{3}$$

$$5 \quad \text{a} \quad p = -\frac{1}{4}, \quad q = 4$$



c

x	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$4 + \frac{1}{x}$	5	$4\frac{2}{3}$	$4\frac{1}{2}$	$4\frac{2}{5}$	$4\frac{1}{3}$

$$\text{area} \approx \frac{1}{2} \times \frac{1}{2} \times [5 + 4\frac{1}{3} + 2(4\frac{2}{3} + 4\frac{1}{2} + 4\frac{2}{5})] \\ = 9\frac{7}{60} \text{ or } 9.12 \text{ (3sf)}$$

$$2 \quad \text{a} \quad \begin{array}{cccccc} x & 0 & 2 & 4 & 6 \\ \sqrt{x^2+4} & 2 & \sqrt{8} & \sqrt{20} & \sqrt{40} \end{array} \\ \text{area} \approx \frac{1}{2} \times 2 \times [2 + \sqrt{40} + 2(\sqrt{8} + \sqrt{20})] \\ = 22.9 \text{ (3sf)}$$

b over-estimate
curve passes below top of each trapezium

$$4 \quad \text{a} \quad 4x^{\frac{1}{2}} - x^{\frac{3}{2}} = 0$$

$$x^{\frac{1}{2}}(4 - x) = 0$$

$$x^{\frac{1}{2}} = 0 \quad [\Rightarrow x = 0, \text{ at } O] \text{ or } x = 4$$

$$\therefore A(4, 0)$$

$$\text{b} \quad \frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$\text{SP: } 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$$

$$\frac{1}{2}x^{-\frac{1}{2}}(4 - 3x) = 0$$

$$x^{-\frac{1}{2}} = 0 \Rightarrow \text{no solutions}$$

$$\therefore x = \frac{4}{3} \text{ at } B$$

$$\text{c} \quad = \int_0^4 (4x^{\frac{1}{2}} - x^{\frac{3}{2}}) \, dx$$

$$= [\frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}]_0^4$$

$$= (\frac{64}{3} - \frac{64}{5}) - 0 = 8\frac{8}{15}$$

$$6 \quad \text{a} \quad 4x - y + 11 = 0 \Rightarrow y = 4x + 11$$

$$\text{intersect when } 2x^2 + 6x + 7 = 4x + 11$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1$$

$$\therefore (-2, 3) \text{ and } (1, 15)$$

b area below curve

$$= \int_{-2}^1 (2x^2 + 6x + 7) \, dx$$

$$= [\frac{2}{3}x^3 + 3x^2 + 7x]_{-2}^1$$

$$= (\frac{2}{3} + 3 + 7) - (-\frac{16}{3} + 12 - 14) = 18$$

area below line

$$= \frac{1}{2} \times 3 \times (3 + 15) = 27$$

area between line and curve

$$= 27 - 18 = 9$$

7 a minimum when $\sin x = 1$

$$\therefore x = \frac{\pi}{2}$$

$$\therefore \left(\frac{\pi}{2}, \frac{1}{2}\right)$$

b	x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
	$\frac{1}{1+\sin x}$	1	0.6667	0.5359

$$\therefore \text{area} \approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0.5359 + 2(0.6667)] = 0.751 \text{ (3sf)}$$

8 a $= 1 + 12\left(\frac{x}{10}\right) + \frac{12 \times 11}{2} \left(\frac{x}{10}\right)^2 + \frac{12 \times 11 \times 10}{3 \times 2} \left(\frac{x}{10}\right)^3 + \dots$

$$= 1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3 + \dots$$

b $\approx \int_0^1 \left(1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3\right) dx$

$$= \left[x + \frac{3}{5}x^2 + \frac{11}{50}x^3 + \frac{11}{200}x^4\right]_0^1$$

$$= \left(1 + \frac{3}{5} + \frac{11}{50} + \frac{11}{200}\right) - 0 = 1\frac{7}{8}$$

9 a at A, $x = 0 \Rightarrow (0, 2)$

$$\frac{dy}{dx} = -1 - 2x$$

grad at A = -1

$$\therefore y = 2 - x$$

b curve cuts x-axis when $y = 0$

$$2 - x - x^2 = 0$$

$$(2 + x)(1 - x) = 0$$

$$x = -2, 1$$

area below curve

$$= \int_0^1 (2 - x - x^2) dx$$

$$= \left[2x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - 0 = \frac{7}{6}$$

tangent cuts x-axis when $y = 0$

$$x = 2$$

area below line

$$= \frac{1}{2} \times 2 \times 2 = 2$$

shaded area

$$= 2 - \frac{7}{6}$$

$$= \frac{5}{6}$$