

1 a at  $A$ ,  $x = 0 \therefore A(0, 4)$

at  $B$ ,  $y = 0$

$$(x^{\frac{1}{2}} - 2)^2 = 0$$

$$x^{\frac{1}{2}} = 2$$

$$x = 4 \therefore B(4, 0)$$

b  $= \int_0^4 (x - 4x^{\frac{1}{2}} + 4) dx$

$$= \left[ \frac{1}{2}x^2 - \frac{8}{3}x^{\frac{3}{2}} + 4x \right]_0^4$$

$$= \left( 8 - \frac{64}{3} + 16 \right) - 0$$

$$= \frac{8}{3}$$

3 a  $4^{x+1} = 32$

$$(2^2)^{x+1} = 2^5$$

$$2x + 2 = 5$$

$$x = \frac{3}{2}$$

b

$x$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
$4^{x+1}$	4	8	16	32

$$\therefore \text{area} \approx \frac{1}{2} \times \frac{1}{2} \times [4 + 32 + 2(8 + 16)]$$

$$= 21$$

2  $= \int_1^2 \left( \frac{3}{2}x + \frac{1}{2}x^{-2} \right) dx$

$$= \left[ \frac{3}{4}x^2 - \frac{1}{2}x^{-1} \right]_1^2$$

$$= \left( 3 - \frac{1}{4} \right) - \left( \frac{3}{4} - \frac{1}{2} \right)$$

$$= \frac{5}{2}$$

4 a at  $A$ ,  $x^2 - 2x = 0$

$$x(x - 2) = 0$$

$$x = 0 \text{ (at } O) \text{ or } 2 \therefore A(2, 0)$$

at  $B$ ,  $x^2 - 2x = x$

$$x(x - 3) = 0$$

$$x = 0 \text{ (at } O) \text{ or } 3 \therefore B(3, 3)$$

b  $\int_0^2 (x^2 - 2x) dx$

$$= \left[ \frac{1}{3}x^3 - x^2 \right]_0^2$$

$$= \left( \frac{8}{3} - 4 \right) - 0 = -\frac{4}{3}$$

$$\therefore \text{area} = \frac{4}{3}$$

c area below curve between  $A$  and  $B$

$$= \int_2^3 (x^2 - 2x) dx$$

$$= \left[ \frac{1}{3}x^3 - x^2 \right]_2^3$$

$$= (9 - 9) - \left( -\frac{4}{3} \right) = \frac{4}{3}$$

area below straight line  $OB$

$$= \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$$

area between curve and line

$$= \frac{9}{2} - \frac{4}{3} + \frac{4}{3}$$

$$= \frac{9}{2}$$

5 a

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.319	1.024	0

$$\text{b } \approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0 + 2(1.319 + 1.024)]$$

$$= 1.49 \text{ (3sf)}$$

c under-estimate  
curve passes above top of each trapezium

$$7 \text{ a } \frac{dy}{dx} = 3x^2 - 6x$$

$$\text{SP: } 3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ (at } P) \text{ or } 2$$

$$\therefore Q(2, 1)$$

$$\text{b } x^3 - 3x^2 + 5 = 5$$

$$x^2(x - 3) = 0$$

$$x = 0 \text{ (at } P) \text{ or } 3$$

$$\therefore R(3, 5)$$

c area below curve

$$= \int_0^3 (x^3 - 3x^2 + 5) \, dx$$

$$= \left[ \frac{1}{4}x^4 - x^3 + 5x \right]_0^3$$

$$= \left( \frac{81}{4} - 27 + 15 \right) - 0 = \frac{33}{4}$$

area below line

$$= 3 \times 5 = 15$$

shaded area

$$= 15 - \frac{33}{4}$$

$$= 6\frac{3}{4}$$

$$6 \int_1^k (3 - 4x^{-2}) \, dx$$

$$= [3x + 4x^{-1}]_1^k$$

$$= \left( 3k + \frac{4}{k} \right) - (3 + 4)$$

$$\therefore 3k + \frac{4}{k} - 7 = 6$$

$$3k^2 - 13k + 4 = 0$$

$$(3k - 1)(k - 4) = 0$$

$$k > 1 \therefore k = 4$$

$$8 \text{ a } (2, 0)$$

$$\text{b } \begin{array}{cccccc} x & 0 & \frac{1}{2} & 1 & \frac{3}{2} & 2 \end{array}$$

$$(2-x)^3 \quad \begin{array}{cccccc} 8 & \frac{27}{8} & 1 & \frac{1}{8} & 0 \end{array}$$

$$\text{area} \approx \frac{1}{2} \times \frac{1}{2} \times [8 + 0 + 2(\frac{27}{8} + 1 + \frac{1}{8})]$$

$$= 4\frac{1}{4}$$

$$\text{c } = 2^3 + 3(2^2)(-x) + 3(2)(-x)^2 + (-x)^3$$

$$= 8 - 12x + 6x^2 - x^3$$

$$\text{d area} = \int_0^2 (8 - 12x + 6x^2 - x^3) \, dx$$

$$= [8x - 6x^2 + 2x^3 - \frac{1}{4}x^4]_0^2$$

$$= (16 - 24 + 16 - 4) - 0$$

$$= 4$$

$$\therefore \% \text{ error} = \frac{4\frac{1}{4} - 4}{4} \times 100\% = 6.25\%$$