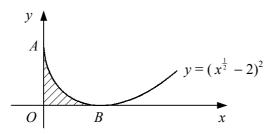
C2

INTEGRATION

Worksheet E

1



The diagram shows the curve with the equation $y = (x^{\frac{1}{2}} - 2)^2$. The curve meets the y-axis at the point A and the x-axis at the point B.

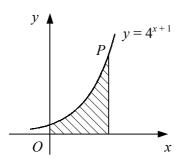
a Find the coordinates of the points A and B. (3)

b Find the area of the shaded region enclosed by the curve and the coordinate axes. (6)

2 Evaluate

$$\int_{1}^{2} \frac{3x^3 + 1}{2x^2} \, \mathrm{d}x. \tag{5}$$

3



The diagram shows the curve with equation $y = 4^{x+1}$.

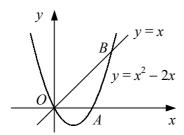
The point *P* on the curve has *y*-coordinate 32.

a Find the x-coordinate of P. (3)

The shaded region is bounded by the curve, the coordinate axes and the line through P parallel to the y-axis.

b Use the trapezium rule with 4 equally-spaced ordinates to estimate the area of the shaded region.(5)

4



The diagram shows the curve $y = x^2 - 2x$ and the line y = x. The curve crosses the x-axis at the origin, O, and at the point A. The line intersects the curve at O and at the point B.

a Find the coordinates of the points A and B. (4)

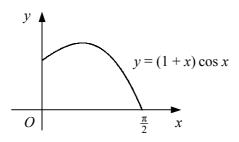
b Find the area of the region enclosed by the curve and the x-axis. (5)

c Show that the area of the region enclosed by the curve and the line y = x is $\frac{9}{2}$. (5)

(3)

(7)

5



The diagram shows the curve with equation $y = (1 + x) \cos x$, $0 \le x \le \frac{\pi}{2}$.

a Copy and complete the table below for points on the curve, giving the *y* values correct to 3 decimal places where appropriate.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
У				

b Use the trapezium rule with the values in your table to estimate the area of the region bounded by the curve and the coordinate axes.(4)

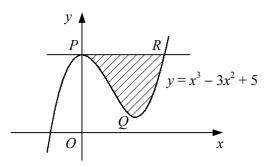
c State, with a reason, whether your answer to part b is an under-estimate or an over-estimate of the true area. (2)

6 Given that

$$\int_{1}^{k} (3 - \frac{4}{x^{2}}) dx = 6,$$

and that k > 1, find the value of the constant k.

7



The diagram shows the curve with the equation $y = x^3 - 3x^2 + 5$. The curve is stationary at the point P(0, 5) and at the point Q.

a Find the coordinates of the point Q. (5)

The straight line passing through the point P parallel to the x-axis intersects the curve again at the point R.

b Find the coordinates of the point R. (2)

 \mathbf{c} Find the area of the shaded region enclosed by the curve and the straight line PR. (7)

8 The finite region R is bounded by the curve $y = (2 - x)^3$ and the coordinate axes.

a State the coordinates of the point where the curve crosses the x-axis. (1)

b Use the trapezium rule with 4 intervals of equal width to estimate the area of R. (5)

c Expand $(2-x)^3$ in ascending powers of x. (2)

d Hence, using integration, find the percentage error in the estimate for the area of R found in part b.(6)