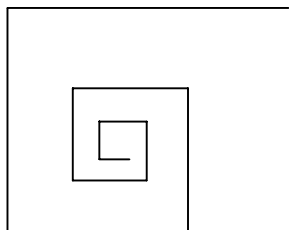**1** The third and fourth terms of a geometric series are 27 and $20\frac{1}{4}$ respectively.

   a Find the first term of the series.

   b Find the sum to infinity of the series.

2 The first three terms of a geometric series are $(k - 8)$, $(k + 4)$ and $(3k + 2)$ respectively, where k is a positive constant.

   a Find the value of k.

   b Find the sixth term of the series.

   c Show that the sum of the first ten terms of the series is 50 857.3 to 1 decimal place.

3 The second and fifth terms of a geometric series are 75 and 129.6 respectively.

   a Show that the first term of the series is 62.5

   b Find the value of the tenth term of the series to 1 decimal place.

   c Find the sum of the first 12 terms of the series to 1 decimal place.

4 **a** Prove that the sum, S_n, of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1 - r^n)}{1 - r}.$$

   b A geometric series has first term 2 and common ratio $\sqrt{2}$.

   Given that the sum of the first n terms of the series is $126(\sqrt{2} + 1)$, find the value of n.

5 The first term of a geometric series is 18 and the sum to infinity of the series is 15.

   a Find the common ratio of the series.

   b Find the third term of the series.

   c Find the exact difference between the sum of the first eight terms of the series and the sum to infinity of the series.

6 The sum of the first n terms of a geometric series is given by $5(3^n - 1)$.

   a Show that the third term of the series is 90.

   b Find an expression for the nth term of the series in the form $k(3^n)$ where k is an exact fraction.

7

A student programs a computer to draw a series of straight lines with each line beginning at the end of the previous one and at right angles to it. The first line is 4 mm long and thereafter each line is 25% longer than the previous one, so that a spiral is formed as shown above.

   a Find the length, in mm, of the eighth straight line drawn by the program.

   b Find the total length of the spiral, in metres, when 20 straight lines have been drawn.

8   The second and fourth terms of a geometric series are 30 and 2.7 respectively.

Given that the common ratio, r, of the series is positive,

   a   find the value of r and the first term of the series,

   b   find the sum to infinity of the series.

9   **a**   Evaluate $\sum\limits_{r=3}^{10} 3^r$.

   b   Show that $\sum\limits_{r=1}^{15} (2^r - 12r) = 64\,094$.

10   A geometric series has common ratio r and the nth term of the series is denoted by u_n.

Given that $u_1 = 64$ and that $u_3 - u_2 = 20$,

   a   show that $16r^2 - 16r - 5 = 0$,

   b   find the two possible values of r,

   c   find the fourth term of the series corresponding to each possible value of r.

   d   Taking the value of r such that the series converges, find the sum to infinity of the series.

11   A geometric series has first term 4 and common ratio $\frac{1}{2}$.

   a   Find the eighth term of the series as an exact fraction.

   b   Find the nth term of the series in the form 2^y where y is a function of n.

   c   Show that the sum of the first n terms of the series is $8 - 2^{3-n}$.

12   The sequence of terms u_1 , u_2 , u_3 , … is defined by

$$u_n = 4 \times 3^n, \quad n \geq 1.$$

   a   Find u_6 .

   b   Find the smallest value of t such that the sum of the first t terms of the sequence is greater than 10^{25}.

13   The sum of the first and third terms of a geometric series is 150. The sum of the second and fourth terms of the series is −75.

   a   Find the first term and common ratio of the series.

   b   Find the sum to infinity of the series.

14   Three consecutive terms of an arithmetic series are a, b and $(3a + 4)$ respectively.

   a   Find an expression for b in terms of a.

Given also that a, b and $(6a + 1)$ respectively are consecutive terms of a geometric series and that a and b are integers,

   b   find the values of a and b.

15   When a ball is dropped onto a horizontal floor it bounces such that it reaches a maximum height of 60% of the height from which it was dropped.

   a   Find the maximum height the ball reaches after its fourth bounce when it is initially dropped from 3 metres above the floor.

   b   Show that when the ball is dropped from a height of h metres above the floor it travels a total distance of $4h$ metres before coming to rest.