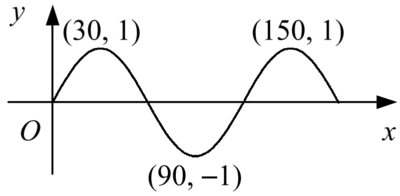
1 **a** $\theta + \frac{\pi}{4} = \pi - 0.4115, 2\pi + 0.4115$
$= 2.7301, 6.6947$
$\theta = 1.94^c, 5.91^c$

b $\cos 2\theta = \frac{1}{3}$
$2\theta = 1.2310, 2\pi - 1.2310$
$2\pi + 1.2310, 4\pi - 1.2310$
$= 1.2310, 5.0522, 7.5141, 11.3354$
$\theta = 0.62^c, 2.53^c, 3.76^c, 5.67^c$

2 **a**



b $(\tan\theta + 1)(\tan\theta - 3) = 0$
$\tan\theta = -1$ or 3
$\theta = 180 - 45, 360 - 45$ or $71.6, 180 + 71.6$
$\theta = 71.6°$ (1dp), $135°$, $251.6°$ (1dp), $315°$

3 **a** $260° = \frac{260}{180}\pi = 4.538$ radians

b $P = (2 \times 6.4) + (6.4 \times 4.538)$
$= 41.8$ cm (3sf)

c $A = \frac{1}{2} \times (6.4)^2 \times 4.538$
$= 92.9$ cm^2 (3sf)

4 $3\cos^2\theta + 6\cos\theta = 2(1 - \cos^2\theta) + 6$
$5\cos^2\theta + 6\cos\theta - 8 = 0$
$(5\cos\theta - 4)(\cos\theta + 2) = 0$
$\cos\theta = 0.8$ or -2 [no solutions]
$\theta = 36.9, 360 - 36.9$
$\theta = 36.9°, 323.1°$

5 **a** area $= \frac{1}{2} \times 4 \times 5 \times \sin 60°$
$= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ cm^2

b $AB^2 = 4^2 + 5^2 - (2 \times 4 \times 5 \times \cos 60°)$
$= 16 + 25 - (40 \times \frac{1}{2}) = 21$
$\therefore AB = \sqrt{21}$ cm

c $\frac{\sin(\angle ABC)}{4} = \frac{\sin 60°}{\sqrt{21}}$
$\therefore \sin(\angle ABC) = \frac{4 \times \frac{\sqrt{3}}{2}}{\sqrt{3}\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{2}{7}\sqrt{7}$

6 $2x + 15 = 63.435, 180 + 63.435,$
$360 + 63.435, 540 + 63.435$
$= 63.435, 243.435, 423.435, 603.435$
$2x = 48.435, 228.435, 408.435, 588.435$
$x = 24.2, 114.2, 204.2, 294.2$

7 $\sin^2\theta - \cos^2\theta = \cos\theta$
$(1 - \cos^2\theta) - \cos^2\theta = \cos\theta$
$2\cos^2\theta + \cos\theta - 1 = 0$
$(2\cos\theta - 1)(\cos\theta + 1) = 0$
$\cos\theta = 0.5$ or -1
$\theta = 60, 360 - 60$ or 180
$\theta = 60°, 180°, 300°$

8 **a** $(x - 5)^2 - 25 + (y - 1)^2 - 1 - 3 = 0$
$(x - 5)^2 + (y - 1)^2 = 29$
$\therefore$ centre $(5, 1)$ radius $\sqrt{29}$

b sub. $x^2 + 36 - 10x - 12 - 3 = 0$
$x^2 - 10x + 21 = 0$
$(x - 3)(x - 7) = 0$
$x = 3, 7$
$\therefore$ $(3, 6)$ and $(7, 6)$

c mid-point of chord $= (5, 6)$
angle of sector $= 2 \times \tan^{-1} \frac{2}{5} = 0.761^c$
area $= \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{29}{2}(0.761 - \sin 0.761) = 1.03$ (3sf)

9 $5\sin^2\theta + 5\sin\theta + 2(1 - \sin^2\theta) = 0$
$3\sin^2\theta + 5\sin\theta + 2 = 0$
$(3\sin\theta + 2)(\sin\theta + 1) = 0$
$\sin\theta = -\frac{2}{3}$ or -1
$\theta = 180 + 41.8,\ 360 - 41.8$ or 270
$\theta = 221.8°$ (1dp), $270°$, $318.2°$ (1dp)

10 **a** $(158°, 0)$, $(338°, 0)$
b $(0, \tan 22°) = (0, 0.404)$ [y-coord to 3sf]
c $x = 68°$ and $x = 248°$

11 **a** $\tan x = 0.4$
$x = 21.8,\ 180 + 21.8$
$x = 21.8°,\ 201.8°$

b $2\sin^2 y - \sin y - 1 = 0$
$(2\sin y + 1)(\sin y - 1) = 0$
$\sin y = -0.5$ or 1
$y = \pi + \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$
$y = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$

12 $3\cos^2\theta - 5\cos\theta + 2(1 - \cos^2\theta) = 0$
$\cos^2\theta - 5\cos\theta + 2 = 0$
$\cos\theta = \frac{5 \pm \sqrt{25-8}}{2}$
$\cos\theta = \frac{1}{2}(5 - \sqrt{17})$ or $\frac{1}{2}(5 + \sqrt{17})$ [no sols]
$\theta = -64.0°,\ 64.0°$

13 **a** $60° = \frac{\pi}{3}$
area $= \frac{1}{2} \times a^2 \times \frac{\pi}{3} = \frac{1}{6}\pi a^2$

b $OC = OA\cos 60° = \frac{1}{2}a$

c area of triangle $OAC = \frac{1}{2} \times a \times \frac{1}{2}a \times \sin 60°$
$= \frac{1}{4}a^2 \times \frac{\sqrt{3}}{2} = \frac{1}{8}a^2\sqrt{3}$

shaded area $= \frac{1}{6}\pi a^2 - \frac{1}{8}a^2\sqrt{3}$
$= \frac{1}{24}a^2(4\pi - 3\sqrt{3})$