1 **a** $x + 40 = \pm 72.5$

$x = -112.5°, 32.5°$

b $\tan 2x = -2$

$2x = 180 - 63.435, 360 - 63.435,$
$-63.435, -180 - 63.435$
$= -243.435, -63.435, 116.565, 296.565$

$x = -121.7°, -31.7°, 58.3°, 148.3°$

2 $\tan x = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \frac{1}{2}\sqrt{2}$

$x = 59.6, 180 + 59.6$ or $16.3, 180 + 16.3$

$x = 16.3, 59.6, 196.3, 239.6$

3 **a** $15\theta = 32.1$

$\theta = 32.1 \div 15 = 2.14$

b $A = \frac{1}{2} \times 15^2 \times 2.14$

$= 240.75 \text{ cm}^2$

4 $2x - \frac{\pi}{3} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}$

$2x = \frac{\pi}{2}, \frac{7\pi}{6}$

$x = \frac{\pi}{4}, \frac{7\pi}{12}$

5 **a** $\sin^2 A = (1 - \sqrt{2})^2$

$= 1 - 2\sqrt{2} + 2 = 3 - 2\sqrt{2}$

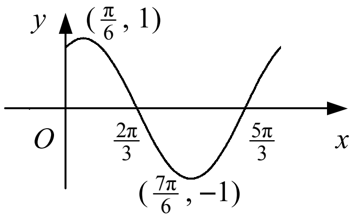
$\cos^2 A = 1 - \sin^2 A = 2\sqrt{2} - 2$

$\therefore \cos^2 A + 2\sin A$

$= 2\sqrt{2} - 2 + 2(1 - \sqrt{2})$

$\therefore \cos^2 A + 2\sin A = 0$

b



6 $2\sin^2 x + \sin x + 1 = 1 - \sin^2 x$

$3\sin^2 x + \sin x = 0$

$\sin x(3\sin x + 1) = 0$

$\sin x = 0$ or $-\frac{1}{3}$

$x = 0, 180, 360$ or $180 + 19.5, 360 - 19.5$

$x = 0, 180°, 199.5°$ (1dp), $340.5°$ (1dp), $360°$

7 **a** $\frac{\sin(\angle PRQ)}{10} = \frac{\sin 0.7}{14}$

$\sin(\angle PRQ) = \frac{10 \times \sin 0.7}{14} = 0.4602$

$\angle PRQ = 0.48^c$

b $\angle PQR = \pi - (0.7 + 0.4782) = 1.963$

area of $\Delta = \frac{1}{2} \times 10 \times 14 \times \sin 1.963$

$= 64.67$

area of sector $= \frac{1}{2} \times 10^2 \times 0.7$

$= 35$

shaded area $= 64.67 - 35$

$= 29.7 \text{ cm}^2$ (3sf)

8 **a** **i** $\cos^2 A = 1 - \sin^2 A = 1 - \frac{5}{9} = \frac{4}{9}$

$\cos A = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$0 < A < 90 \;\therefore \cos A = \frac{2}{3}$

ii $\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{5}}{3} \div \frac{2}{3} = \frac{1}{2}\sqrt{5}$

b $\cos x(5\sin x + 1) = 0$

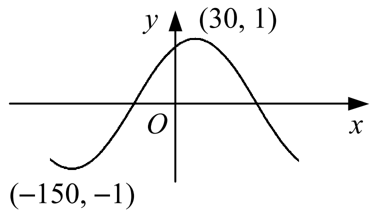
$\cos x = 0$ or $\sin x = -0.2$

$x = 90, 270$ or $180 + 11.5, 360 - 11.5$

$x = 90°, 191.5°$ (1dp), $270°, 348.5°$ (1dp)

9 $2\theta + 30 = 180 - 60,\ 180 + 60$
$= 120,\ 240$
$2\theta = 90,\ 210$
$\theta = 45,\ 105$

10 **a**



b $\cos(x - 30) = 0.2$
$x - 30 = \pm 78.5$
$x = -48.5,\ 108.5$

11 $4\cos^2 x - \cos x - 2(1 - \cos^2 x) = 0$
$6\cos^2 x - \cos x - 2 = 0$
$(3\cos x - 2)(2\cos x + 1) = 0$
$\cos x = \frac{2}{3}$ or -0.5
$x = 48.2,\ 360 - 48.2$ or $180 - 60,\ 180 + 60$
$x = 48.2°$ (1dp), $120°$, $240°$, $311.8°$ (1dp)

12 **a** area of sector $= \frac{1}{2} \times r^2 \times \theta$
area of triangle $= \frac{1}{2} \times r^2 \times \sin\theta$
$A_1 = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$
$= \frac{1}{2}r^2(\theta - \sin\theta)$ cm^2

b $\theta = \frac{5\pi}{6}$ $\therefore A_1 = \frac{1}{2}r^2(\frac{5\pi}{6} - \frac{1}{2})$
$= \frac{1}{12}r^2(5\pi - 3)$
$A_2 = \pi r^2 - A_1 = \pi r^2 - (\frac{5}{12}\pi r^2 - \frac{1}{4}r^2)$
$= \frac{7}{12}\pi r^2 + \frac{1}{4}r^2$
$= \frac{1}{12}r^2(7\pi + 3)$
$\therefore A_1 : A_2 = \frac{1}{12}r^2(5\pi - 3) : \frac{1}{12}r^2(7\pi + 3)$
$= (5\pi - 3) : (7\pi + 3)$

13 $3\sin x - 2\cos^2 x = 0$
$3\sin x - 2(1 - \sin^2 x) = 0$
$2\sin^2 x + 3\sin x - 2 = 0$
$(2\sin x - 1)(\sin x + 2) = 0$
$\sin x = 0.5$ or -2 [no solutions]
$x = \frac{\pi}{6},\ \pi - \frac{\pi}{6}$
$x = \frac{\pi}{6},\ \frac{5\pi}{6}$

14 **a** $7^2 = 5^2 + 8^2 - [2 \times 5 \times 8 \times \cos(\angle ABC)]$
$\cos(\angle ABC) = \frac{25 + 64 - 49}{80}$
$= \frac{1}{2}$

b $\sin(\angle ABC) = \sqrt{1 - (\frac{1}{2})^2} = \frac{\sqrt{3}}{2}$
area $= \frac{1}{2} \times 5 \times 8 \times \frac{\sqrt{3}}{2}$
$= 10\sqrt{3}$

15 **a** LHS $= 2 + 2\tan^2\theta + \cos^2\theta + \sin^2\theta$
$= 2 + 2\tan^2\theta + 1$
$= 3 + 2\tan^2\theta$
$=$ RHS

b $3 + 2\tan^2\theta = 7$
$\tan^2\theta = 2$
$\tan\theta = \pm\sqrt{2}$
$\theta = 54.7,\ 180 + 54.7$
or $180 - 54.7,\ 360 - 54.7$
$\theta = 54.7°,\ 125.3°,\ 234.7°,\ 305.3°$ (1dp)