Godalming College

Mechanics 10 – Kinematics 2: Integration

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Evaluate the following definite integrals.

(i) $\int_{-1}^{1}(4x+5)\mathrm{d}x$ (ii) $\int_{-1}^{0}(6x^2-2x)\mathrm{d}x$ (iii) $\int_{2}^{4}(x^2-x+3)\mathrm{d}x$

(iv) $\int_{-1}^{2}(2+x-x^2)\,\mathrm{d}x$ (v) $\int_{-1}^{2}(x^3-x+4)\mathrm{d}x$ (vi) $\int_{1}^{3}\frac{1}{x^3}\mathrm{d}x$

(vii) $\int_{1}^{9}\frac{1}{\sqrt{x}}\mathrm{d}x$ (viii) $\int_{1}^{4}\frac{2-x+3x^2}{\sqrt{x}}\mathrm{d}x$ (ix) $\int_{1}^{2}\frac{x^2-1}{x^4}\mathrm{d}x$

Section 2 – Consolidation of this week's topic. Please complete all questions.

1. A particle is moving along the x-axis. At time t seconds (where $t \geq 0$), the velocity of P is v ms^{-1} in the direction of x increasing, where $v = 12 - t - t^2$. Find the acceleration of P when P is instantaneously at rest.

(6 marks)

2. At time t seconds, where $t \geq 0$, the velocity v ms^{-1} of a particle moving in a straight line is given by $v = 12 + t - 6t^2$. When $t = 0$, P is at a point O on the line. Find
   (a) The magnitude of the acceleration of P when $v = 0$
   (b) The distance of P from O when $v = 0$

(6+6 = 12 marks)

3. A particle P is moving on the x-axis. At time t seconds, the velocity of P is $(4t\text{-}t^2)$ ms^{-1} in the direction of x increasing. At time $t = 0$, P is at the origin O. Find
   (a) The value of x when $t > 0$ and P is at rest
   (b) The total distance moved by P in the interval $0 \leq t \leq 5$.

(5+5 = 10 marks)

4. A particle P moves in a straight line so that, at time t seconds, its velocity v ms^{-1} is given by

$$v = \begin{cases} 4, & 0 \leq t \leq 2 \\ 5 - \frac{4}{t^2}, & t > 2 \end{cases}$$

   (a) Sketch a velocity-time graph to illustrate the motion of P.
   (b) Find the distance moved by P in the interval $0 < t \leq 5$.

(3+5 = 8 marks)

5. A particle P moves in a straight line so that, at time t seconds, its acceleration a ms^{-2} is given by $a = \begin{cases} 6t - t^2, & 0 \leq t \leq 2 \\ 8 - t, & t > 2 \end{cases}$

When t = 0 the particle is at a fixed point O on the line. Find
   (a) The speed of P when $t = 2$,
   (b) The speed of P when $t = 4$,
   (c) The distance from O to P when $t = 4$.

(5+5+7 = 17 marks)

(Total 55 Marks)

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

A particle P moves along the x-axis in a straight line so that, at time t seconds, the velocity of P is v m s^{-1}, where

$$v = \begin{cases} 10t - 2t^2, & 0 \le t \le 6, \\ \dfrac{-432}{t^2}, & t > 6. \end{cases}$$

At $t = 0$, P is at the origin O. Find the displacement of P from O when

(a) $t = 6$,

(b) $t = 10$.