Mechanics 24 – Variable Acceleration in 1-d

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.
Please complete all questions.

1)

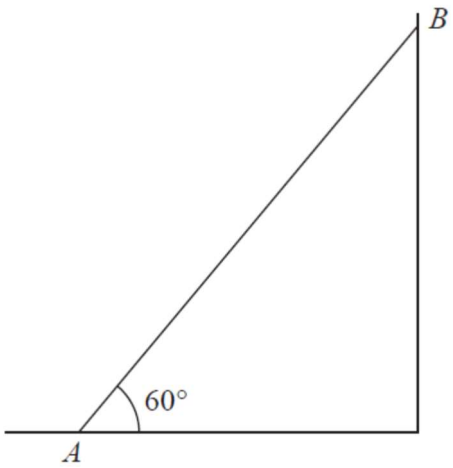

Figure 1

A non-uniform rod, *AB*, of mass *m* and length 2*l*, rests in equilibrium with one end *A* on a rough horizontal floor and the other end *B* against a rough vertical wall. The rod is in a vertical plane perpendicular to the wall and makes an angle of 60° with the floor as shown in Figure 1. The coefficient of friction between the rod and the floor is $\frac{1}{4}$ and the coefficient of friction between the rod and the wall is $\frac{2}{3}$. The rod is on the point of slipping at both ends.

(*a*) Find the magnitude of the vertical component of the force exerted on the rod by the floor.

The centre of mass of the rod is at *G*.

(*b*) Find the distance *AG*.

2) [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

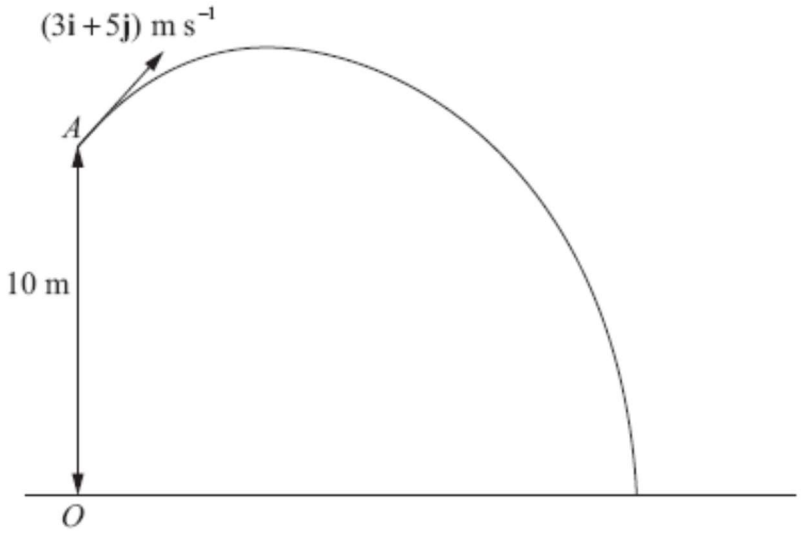

Figure 3

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after T seconds.

(*a*) For $0 \leq t \leq T$, show that, with respect to O, the position vector, **r** metres, of P at time t seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$$

(*b*) Find the value of T.

(*c*) Find the velocity of P at time t seconds ($0 \leq t \leq T$).

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(*d*) Find the time taken for P to move from A to B.

(*e*) Find the speed of P as it passes through B.

3) Michael is using the large data set to investigate the relationship between the time of the year and the maximum daily temperature, T °C, in the UK.
He looks at the daily temperatures in Leuchars to do this.
Starting with 01/05/2015, he labels each of the days in the large data set with a number x. The day 01/05/2015 is given the number 1, the day 02/05/2015 is given the number 2 and so on.

He then plots a scatter diagram of T against x.
Michael expects there to be 184 values for x.

(a) Using your knowledge of the large data set, explain why
(i) he expects there to be 184 values for x,
(ii) there may be less than 184 values of x.

Michael calculates the regression line for T on x.
His regression line has the equation

$$T = 16.551 - 0.0027x$$

(b) Interpret the gradient of Michael's regression line.
(c) Estimate the temperature in Leuchars on 03/05/2015.
(d) Use your knowledge of the large data set to explain why it is unreliable to use Michael's regression line to estimate the temperature on days in Leuchars.

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Section 2 – Consolidation of this week's topic.
Please complete all questions.

1) A particle's displacement from the origin O in metres at t seconds is given by

$$s = 2t^3 - 4t^2 + 3$$

Find
a) the displacement from O when t = 3 **(1)**
b) an expression for the velocity at t seconds **(3)**
c) the velocity when t = 3 **(1)**

2) A particle's displacement from the origin O in metres at t seconds is given by

$$s = e^{2t}$$

Find
a) the velocity when t = 0.4 **(3)**
b) the acceleration when t = 0.3 **(3)**

3) A particle's displacement from the origin O in metres at t seconds is given by

$$s = \sin\left(\frac{\pi t}{6}\right)$$

Find
a) the velocity when t = 1 **(3)**
b) the acceleration when t = 2 **(3)**

4) A particle P moves on the x-axis starting at the origin. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geq 0$$

Find

(a) the times when P is instantaneously at rest, **(2)**

(b) the greatest speed of P in the interval $0 \leq t \leq 4$, **(4)**

(c) the total distance travelled by P in the interval $0 \leq t \leq 4$.
(answer to the nearest metre) **(4)**

5) The acceleration in ms^{-2} of a particle moving on the x-axis (scale in metres) is given by

$$a = (2t + 1)^{\frac{1}{2}}$$

Initially the particle is at x =4 and its velocity is $2ms^{-1}$ in the positive direction.

Find a) the speed when t = 4 **(3)**

b) the position on the x-axis when t = 12 **(4)**

6) A bird is flying in a straight line. For the first 20 seconds of its flight, its velocity is modelled by

$$v = 0.1t^{\frac{3}{2}}$$

After 20 seconds, its velocity is modelled by

$$v = 0.1(40 - t)^{\frac{3}{2}}$$

a) Find the distance travelled by the bird after i) 5 sec **(3)**

ii) 20 sec **(3)**

iii) 32 sec **(4)**

b) State a limitation of the model. **(1)**

Total : 45 marks

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