Godalming College

Mechanics 6 – Vectors: Solutions

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1

1. (a) 3g – 12 = 3a, so a = 5.8
   (b) 120 – 6g = 6a, so a = 10.2
   (c) 5 + 11 – 1.5g = 1.5a, so a = 0.867 (3 d.p.)

2. (i) 12g – T = 12 x 2.5, so T = 87.6 N
   (ii) 12g – T = 12 x 0, so T = 12g or 118 N (3 s.f.)
   (iii) 12g – T = 12 x -1.5, so T = 136 N (3 s.f.)

Section 2 – Consolidation of this week's topic. Please complete all questions.

1. $F_1 + F_2 + F_3 = 0 \Rightarrow 3 + b + 2 = 0$ and $a + (-7) + 6 = 0 \Rightarrow a = 1,\ b = -5$ ✓ (4 marks)

2. $\sqrt{(56a)^2 + (-42a)^2} = \sqrt{(4900a)^2} = 70a = 35 \Rightarrow a = 0.5$ ✓ (3 marks)

3. (a) $\begin{pmatrix}3\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\end{pmatrix} = k\begin{pmatrix}1\\1\end{pmatrix} \Rightarrow 3 + \lambda = k$ and $4 - 2\lambda = k \Rightarrow 3 + \lambda = 4 - 2\lambda$
   $\Rightarrow \lambda = \frac{1}{3}$ ✓

   (b) $\mu\begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}1\\-2\end{pmatrix} = k\begin{pmatrix}1\\3\end{pmatrix} \Rightarrow 3\mu + 1 = k$ and $4\mu - 2 = 3k \Rightarrow 9\mu + 3 = 4\mu - 2$
   $\Rightarrow \mu = -1$ ✓ (8 marks)

4. (a) Bearing = $\arctan\left(\frac{24}{7}\right) = 254°$ (nearest degree) ✓
   (b) $\sqrt{(24a)^2 + (7a)^2} = \sqrt{(625a)^2} = -25a = 200 \Rightarrow a = -8$ ✓ (5marks)

5. (a) Resultant force $= F_1 + F_2 + F_3 = ma \Rightarrow \begin{pmatrix}p\\-8\end{pmatrix} + \begin{pmatrix}5\\q\end{pmatrix} + \begin{pmatrix}6\\-19\end{pmatrix} = \frac{2}{3}\begin{pmatrix}36\\-15\end{pmatrix} = \begin{pmatrix}24\\-10\end{pmatrix}$ ✓
   $\Rightarrow p + 11 = 24$ and $-q - 27 = -10$
   $\Rightarrow p = 13$ and $q = 17$
   (b) Magnitude $= \sqrt{(24)^2 + (-10)^2} = \sqrt{676} = 26$ N ✓
   (c) Angle $= \arctan\left(\frac{10}{24}\right) = (-)22.6°$ (to 1 d.p.) ✓
   (d) $u = \begin{pmatrix}4\\1\end{pmatrix}, a = \begin{pmatrix}36\\-15\end{pmatrix}, t = \frac{1}{3} \Rightarrow v = u + at = \begin{pmatrix}4\\1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}36\\-15\end{pmatrix} = \begin{pmatrix}16\\-4\end{pmatrix}$ ✓. (12 marks)

6. (a) $u = \begin{pmatrix} 8 \\ -5 \end{pmatrix}, v = \begin{pmatrix} -1 \\ 7 \end{pmatrix}, t = 6 \Rightarrow v = u + at \Rightarrow \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 \\ -5 \end{pmatrix} + 6a \Rightarrow a = \begin{pmatrix} -1.5 \\ 2 \end{pmatrix}$ ✓

(b) $F = ma = 0.7\begin{pmatrix} -1.5 \\ 2 \end{pmatrix} = \begin{pmatrix} -1.05 \\ 1.4 \end{pmatrix}$ ✓

Magnitude $= \sqrt{(-1.05)^2 + (1.4)^2} = \sqrt{3.0625} = 1.75$ N ✓ (7 marks)

7. $a = \begin{pmatrix} 9 \\ -3 \\ 12 \end{pmatrix}, b = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ so the resultant $= \begin{pmatrix} 9+x \\ -3+y \\ 12+z \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda - 15 \\ 2\lambda \end{pmatrix}$ ✓

$\Rightarrow x = \lambda - 9,\ y = \lambda - 12,\ z = 2\lambda - 12$ ✓✓

Magnitude of resultant force $= \sqrt{\lambda^2 + (\lambda - 15)^2 + (2\lambda)^2} = 5\sqrt{21}$ ✓

$\Rightarrow \lambda^2 + \lambda^2 - 30\lambda + 225 + 4\lambda^2 = 525$

$\Rightarrow 6\lambda^2 - 30\lambda - 300 = 0$ ✓

$\lambda > 0 \Rightarrow \lambda = 10$ ✓ and $\Rightarrow x = 1, y = -2, z = 8$ ✓ ✓ ✓

(9 marks)

8. (a) $F = ma \Rightarrow a = {}^{F}/_{1.5} = \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix}$ ✓ ✓

(b) $\cos\theta_x = {}^{4}/_{\sqrt{56}} \Rightarrow \theta_x = 57.7^{o}$ ✓ ✓ (1 d.p.)

(c) Let t=0 when velocity $= \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix}$; acceleration is constant so use suvat with

$s = ?, \quad u = \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix}, \quad v = -, \quad a = \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix}, \quad t = 2$ ✓

$s = ut + \frac{1}{2}at^2 = \begin{pmatrix} 4 \\ 18 \\ 10 \end{pmatrix}$ ✓ ✓

(7 marks)

(Total 55 Marks)

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Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

1. The forces $\begin{pmatrix}3\\4\end{pmatrix}$ N and $\begin{pmatrix}4\\3\end{pmatrix}$ N act at the same point. Find the angle between these two forces. Angle = arctan(4/3)-arctan(3/4) = 16.3 degrees (1 d.p)

Exam Q M1 May 2013

2. [*In this question, the horizontal unit vectors* **i** *and* **j** *are directed due east and due north respectively.*]

The velocity, **v** m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}.$$

(*a*) Find the speed of P when $t = 0$.

(*b*) Find the bearing on which P is moving when $t = 2$.

(*c*) Find the value of t when P is moving

(i) parallel to **j**,

(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

Question Number	Scheme	Marks
2.		
(a)	$t = 0$ gives $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$	B1
	speed $= \sqrt{1^2 + (-3)^2}$	M1
	$= \sqrt{10} = 3.2$ or better	A1
		(3)
(b)	$t = 2$ gives $\mathbf{v} = (-3\mathbf{i} + 3\mathbf{j})$	M1
	Bearing is $315°$	A1
		(2)
(c)(i)	$1 - 2t = 0 \Rightarrow t = 0.5$	M1 A1
(ii)	$-(3t - 3) = -3(1 - 2t)$	M1 A1
	Solving for t	**DM**1

Question Number	Scheme	Marks
	t = 2/3, 0.67 or better	A1 **(6)** **[11]**