

Mechanics 9 – Kinematics 1 - Solutions

Section 1

1. (i) $f'(x) = 2$ (ii) $f'(x) = 3x^2 - 5$ (iii) $f'(x) = -3x^4$ (iv) $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$
 (v) $f'(x) = -2x^2 + 6x^{-3}$ (vi) $f'(x) = 2x^{-\frac{1}{2}} + \frac{3}{2}x^{-\frac{3}{2}}$ (vii) $f'(x) = -15x^{-6} + 14x^{-8}$
 (viii) $f'(x) = \frac{4}{3}x^{-\frac{1}{3}} + \frac{10}{3}x^{-\frac{5}{3}}$ (ix) $f'(x) = 12x^3 - 6x^{\frac{1}{2}} - x^{-2}$ (x) $f'(x) = 2x + 2$
2. (i) $\frac{dy}{dx} = 12 - 3x^2 \Rightarrow$ at $x = 0$ $\frac{dy}{dx} = 12$
 (ii) $\frac{dy}{dx} = 12 - 3x^2 = 0 \Rightarrow x = 2$ or $-2 \Rightarrow$ coordinates are $(2, 16)$ and $(-2, -16)$

Section 2

1. $s = t^3 + 2t^2 + 3t + 4$

$$v = \frac{ds}{dt} = 3t^2 + 4t + 3$$

$$a = \frac{dv}{dt} = 6t + 4$$

When $t = 2$, $v = 3 \times 2^2 + 4 \times 2 + 3 = 23$ ✓

$$a = 6 \times 2 + 4 = 16$$
 ✓

(5 marks)

2. (i) When $t = 0$, $s = -2$, so the initial displacement = -2 m. ✓

$$s = 2t^2 + 3t - 2$$

$$v = \frac{ds}{dt} = 4t + 3$$
 ✓

When $t = 0$, $v = 3$ so the initial velocity is 3 ms^{-1} . ✓

(ii) $v = 0 \Rightarrow 4t + 3 = 0 \Rightarrow t = -0.75$ ✓

Since this is negative, there are no times for which the velocity is zero. ✓

(iii) When $s = 0$, $2t^2 + 3t - 2 = 0$ ✓

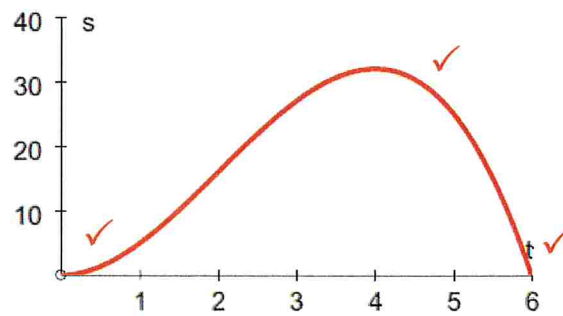
$$(2t - 1)(t + 2) = 0$$
 ✓

$$t = \frac{1}{2} \text{ or } -2$$

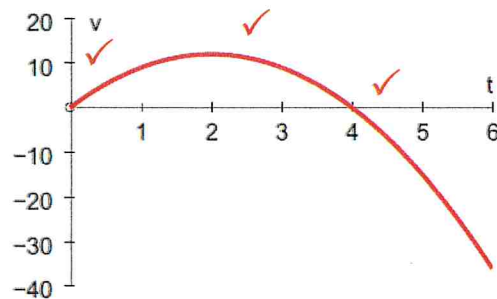
The particle is at the origin when $t = \frac{1}{2}$. ✓

(11 marks)

3. (i) $s = 6t^2 - t^3 = t^2(6 - t)$



$$v = \frac{ds}{dt} = 12t - 3t^2 = 3t(4 - t)$$



(ii) The particle is at 0 when $t = 0$ and when $t = 6$.

(iii) The greatest displacement is when the velocity is zero.

$$3t(4 - t) = 0$$

$$t = 0 \text{ or } t = 4$$

From the graph, the greatest displacement is when $t = 4$

$$s = 4^2(6 - 4) = 32$$

The greatest displacement is 32 m.

(iv) The greatest positive speed is when the acceleration is zero

$$a = \frac{dv}{dt} = 12 - 6t$$

$$\text{When } a = 0, t = 2$$

$$\text{When } t = 2, v = 3 \times 2(4 - 2) = 12$$

The greatest negative speed in the time interval is when $t = 6$ (from the graph).

$$\text{When } t = 6, v = 3 \times 6(4 - 6) = -36$$

So the greatest speed in the time interval is 36 ms^{-1} .

(18 marks)

4. Maximum velocity occurs when acceleration = 0

$$(a) \quad v = \frac{ds}{dt} = 15 + 12t - 3t^2 \quad \checkmark$$

$$a = \frac{dv}{dt} = 12 - 6t = 0 \quad \checkmark \Rightarrow t = 2 \quad \checkmark$$

$$\frac{d^2v}{dt^2} < 0 \Rightarrow \text{maximum} \quad \checkmark$$

$$\Rightarrow \text{maximum velocity} = 15 + 12 \times 2 - 3 \times 2^2 = 27 \text{ms}^{-1}. \quad \checkmark$$

$$(b) \quad v = \frac{ds}{dt} = \frac{-t^4}{4} + t^3 - t^2 + 11 \quad \checkmark$$

$$a = \frac{dv}{dt} = -t^3 + 3t^2 - 2t = 0 \Rightarrow -t(t-1)(t-2) = 0$$

$$\Rightarrow t = 0, 1 \text{ or } 2 \quad \checkmark$$

$$\frac{d^2v}{dt^2} < 0 \text{ when } t = 0 \text{ or } 2 \Rightarrow \text{maximum}$$

$$\Rightarrow \text{maximum velocity} = 11 \text{ms}^{-1} \quad \checkmark$$

(10 marks)

$$5. \quad v = \frac{ds}{dt} = \frac{2}{3}t^3 - 6t^2 + 10t + 2 \quad \checkmark$$

$$a = \frac{dv}{dt} = 2t^2 - 12t + 10 = 0 \Rightarrow 2(t-1)(t-5) = 0$$

$$\Rightarrow t = 1 \text{ or } 5 \quad \checkmark$$

$$v_1 = \frac{20}{3}, \quad v_5 = \frac{-44}{3} \quad \checkmark$$

$$\Rightarrow \text{maximum speed} = \frac{44}{3} \text{ms}^{-1} \quad \checkmark$$

(6 marks)

(Total 50 Marks)

Section 3

$$v = 3t^2 - 4t + c \rightarrow s = t^3 - 2t^2 + ct + d \quad s = 0, t = 0 \text{ so } d = 0$$

$$s = 0, t = 1 \text{ so } c = 1$$

Max s when $v = 0$ so $t = 1/3$ or 1 (we know $s = 0$ when $t = 1$) so max displacement when $t = 1/3$, $s = 4/27$ metres.