

Mechanics 10 – Kinematics 2 - Solutions

Section 1

1. (i) 10 (ii) 3 (iii) $\frac{56}{3}$ (iv) $\frac{9}{2}$ (v) $\frac{57}{4}$ (vi) $\frac{4}{9}$
 (vii) 4 (viii) 36.53 (2 d.p.) (ix) $\frac{5}{24}$

Section 2

1. P is at rest when $v = 0$

$$0 = 12 - t - t^2 \quad \checkmark$$

$$t^2 + t - 12 = (t+4)(t-3) = 0 \quad \checkmark$$

$$t = -4, 3 \quad \checkmark \checkmark$$

As $t \geq 0$, $t = -4$ is rejected. \checkmark

$$a = \frac{dv}{dt} = -1 - 2t \quad \checkmark$$

When $t = 3$,

$$a = -1 - 2 \times 3 = -7 \quad \checkmark$$

The acceleration of P when P comes to instantaneously to rest is 7 m s^{-2} in the direction of x decreasing. \checkmark

(8 marks)

2. (a) $a = \frac{dv}{dt} = 1 - 12t \quad \checkmark$
 When $v = 0$, $12 + t - 6t^2 = 0 \Rightarrow (4 + 3t)(3 - 2t) = 0$
 $t \geq 0$ and so $t = \frac{3}{2} \quad \checkmark \Rightarrow \text{acceleration} = 1 - 12\left(\frac{3}{2}\right) = -17 \quad \checkmark$
 So the magnitude of the acceleration is $17 \text{ ms}^{-2} \quad \checkmark$

(b) $s = \int v \cdot dt = \int 12 + t - t^2 \cdot dt = 12t + \frac{1}{2}t^2 - 2t^3 + c \quad \checkmark$

But $s = 0$ when $t = 0$ and so $c = 0 \quad \checkmark$

So when $v = 0$, $t = \frac{3}{2} \quad \checkmark$

$$\Rightarrow s = 12\left(\frac{3}{2}\right) + \frac{1}{2}\left(\frac{3}{2}\right)^2 - 2\left(\frac{3}{2}\right)^3 \quad \checkmark$$

$$\Rightarrow s = 12.375$$

$$= 12.4 \text{ m (3sf)} \quad \checkmark$$

(12 marks)

3. a P is at rest when $v = 0$

$$v = 4t - t^2 = 0 \quad \checkmark$$

$$t(4-t) = 0$$

As $t > 0$, $t = 4 \quad \checkmark$

$$x = \int v dt$$

$$= 2t^2 - \frac{1}{3}t^3 + c \quad \checkmark$$

When $t = 0, x = 0$

$$0 = 0 - 0 + c = 0 \Rightarrow c = 0 \quad \checkmark$$

$$x = 2t^2 - \frac{1}{3}t^3 \quad \checkmark$$

When $t = 4$

$$x = 2 \times 4^2 - \frac{4^3}{3} = 10\frac{2}{3} \quad \checkmark$$

- b When $t = 5$,

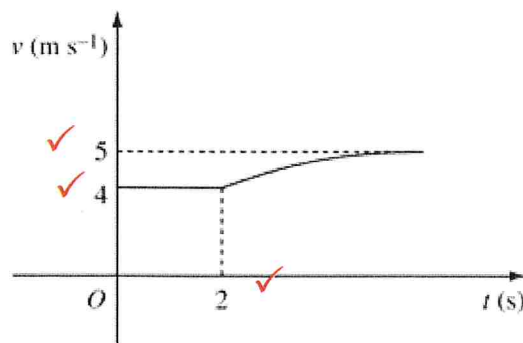
$$x = 2 \times 5^2 - \frac{5^3}{3} = 8\frac{1}{3} \quad \checkmark$$

In the interval $0 \leq t \leq 5$, moves to a point $10\frac{2}{3}$ m from O and then returns to a point $8\frac{1}{3}$ m from O . \checkmark

The total distance moved is $10\frac{2}{3} + (10\frac{2}{3} - 8\frac{1}{3}) = 13$ m. \checkmark

(10 marks)

4. a



- b In the first two seconds P moves $2 \times 4 = 8$ m \checkmark

$$s = \int v dt = \int (5 - 4t^{-2}) dt$$

$$= 5t - \frac{4t^{-2}}{-1} + C = 5t + \frac{4}{t} + C \quad \checkmark$$

When $t = 2, s = 8$

$$8 = 5 \times 2 + \frac{4}{2} + C = 12 + C \Rightarrow C = -4 \quad \checkmark$$

$$s = 5t + \frac{4}{t} - 4$$

When $t = 5$,

$$s = 5 \times 5 + \frac{4}{5} - 4 = 21.8 \quad \checkmark$$

In the interval $0 \leq t \leq 5$, P moves 21.8 m. \checkmark

(8 marks)

5. a For $0 \leq t \leq 2$

$$v = \int a \, dt = \int (6t - t^2) \, dt \quad \checkmark$$
$$= 3t^2 - \frac{1}{3}t^3 + c, \text{ where } c \text{ is a constant of integration.}$$

When $t = 0, v = 0$

$$0 = 0 - 0 + c \Rightarrow c = 0 \quad \checkmark$$

$$v = 3t^2 - \frac{1}{3}t^3 \quad \checkmark$$

When $t = 2,$

$$v = 3 \times 2^2 - \frac{2^3}{3} = \frac{28}{3}$$

The speed of P when $t = 2$ is $\frac{28}{3} \text{ m s}^{-1}$. \checkmark

- b For $t > 2,$

$$v = \int a \, dt = \int (8 - t) \, dt$$
$$= 8t - \frac{1}{2}t^2 + k, \text{ where } k \text{ is a constant of integration.}$$

From a, when $t = 2, v = \frac{28}{3}$ \checkmark

$$\frac{28}{3} = 16 - \frac{4}{2} + k \Rightarrow k = -\frac{14}{3} \quad \checkmark$$

$$v = 8t - \frac{1}{2}t^2 - \frac{14}{3} \quad \checkmark$$

When $t = 4,$

$$v = 32 - 8 - \frac{14}{3} = \frac{58}{3}$$

The speed of P when $t = 4$ is $\frac{58}{3} \text{ m s}^{-1}$. \checkmark

c For $0 \leq t \leq 2$,

$$x = \int v dt = \int \left(3t^2 - \frac{1}{3}t^3 \right) dt = t^3 - \frac{1}{12}t^4 + l, \text{ where } l \text{ is a constant of integration.}$$

When $t = 0, x = 0$

$$0 = 0 - 0 + l \Rightarrow l = 0$$

When $t = 2$,

$$x = 2^3 - \frac{2^4}{12} = \frac{20}{3} \quad (1)$$

For $t > 2$,

$$x = \int v dt = \int \left(8t - \frac{1}{2}t^2 - \frac{14}{3} \right) dt = 4t^2 - \frac{1}{6}t^3 - \frac{14}{3}t + m,$$

where m is a constant of integration.

From (1) above

$$\text{When } t = 2, x = \frac{20}{3}$$

$$\frac{20}{3} = 16 - \frac{8}{6} - \frac{28}{3} + m \Rightarrow m = \frac{4}{3}$$

$$x = 4t^2 - \frac{1}{6}t^3 - \frac{14}{3}t + \frac{4}{3}$$

When $t = 4$,

$$x = 64 - \frac{64}{6} - \frac{56}{3} + \frac{4}{3} = 36$$

The distance from O to P when $t = 4$ is 36 m.

(17 marks)

(Total 55 Marks)

Section 3

<p>(a) $v = 10t - 2t^2, s = \int v dt$ $= 5t^2 - \frac{2t^3}{3} (+C)$ $t = 6 \Rightarrow s = 180 - 144 = 36 \text{ (m)}$</p>	<p>M1 A1 A1 (3)</p>
<p>(b) $s = \int v dt = \frac{-432t^{-1}}{-1} (+K) = \frac{432}{t} (+K)$ $t = 6, s = "36" \Rightarrow 36 = \frac{432}{6} + K$ $\Rightarrow K = -36$ At $t = 10, s = \frac{432}{10} - 36 = 7.2 \text{ (m)}$</p>	<p>B1 M1* A1 d*M1 A1 (5) [8]</p>