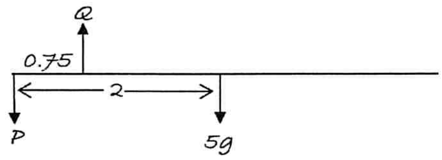
Mechanics 8 – Projectiles 2 : Solutions

Section 1

1. (a) Resolving vertically, $10 + 20 - P = 0$ so $\mathbf{P = 30}$ **N**
   Taking moments from left-hand end, $20 \times 0.6 - P \times x = 0$ so $\mathbf{x = 0.4}$ **m**

   (b) Taking moments from left-hand end, $0.6 \times Q - 0.2 \times 30 = 0$ so $\mathbf{Q = 10}$ **N**
   Resolving vertically, $P + Q - 30 = 0$ so $\mathbf{P = 20}$ **N**

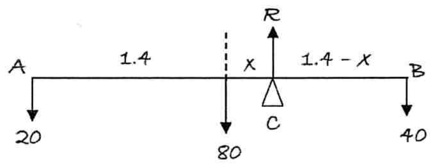
2.



Taking moments from left-hand end, $0.75Q - 5g \times 2 = 0$ so **Q =** $\mathbf{\frac{40g}{3}}$ **N upwards**

Resolving vertically, $Q - P - 5g = 0$ so **P =** $\mathbf{\frac{25g}{3}}$ **N downwards**

3.



Taking moments about C: $20(1.4 + x) + 80x - 40(1.4 - x) = 0$ so $x = 0.2$
Distance of C from A = 1.4 + 0.2 = **1.6 m**

4. Resultant force $= F_1 + F_2 + F_3 = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{i} + 3\mathbf{j} = 3\mathbf{i} + 5\mathbf{j}$ = mass x acceleration
   Mass = 0.1 kg so acceleration $= 30\mathbf{i} + 50\mathbf{j}$ with magnitude $\sqrt{30^2 + 50^2} = \mathbf{10\sqrt{34}}$ **N**
   Direction = $\arctan \frac{50}{30} = \mathbf{59^0}$ to the **i** vector

5.
   (a) 184/17 = 10.82.. so expected sample size will be 10 or 11 depending on starting point but some data may be unavailable.
   (b) Values are legitimate but higher than expected, indicating that some of the very few high values have been included in this sample. Further systematic samples from different start points would probably give lower values.

Section 2

1. Vertically, at ground level s = 0, = = 40sin30, v = ?, a = -g, t = ? ✓
   Using $s = ut + \frac{1}{2}at^2$ we get $0 = 20t - 4.9t^2$ ✓
   So $t = 0$ (start) or $t = \frac{20}{4.9}$ ✓
   Horizontal distance = $40\cos 30 \times t = 40\cos 30 \times \frac{20}{4.9}$ = 141 m (3sf) ✓ **(5 marks)**

2. Vertically, at ground level and taking up as positive s = -2.5, u = 20sin40, v = ?, a = -g, t = ? ✓
   Using $s = ut + \frac{1}{2}at^2$ we get $-2.5 = 20t \times \sin 40 - 4.9t^2$ ✓
   $0 = 4.9t^2 - 20t \times \sin 40 - 2.5 = 0$ ✓
   So $t = 2.81$seconds (3sf) (ignore negative value) ✓ **(4 marks)**

3.
   (a) At top of flight, for vertical motion $s = 10, u = 20 \times \sin\alpha, v = 0, a = -g, t = ?$ ✓
   Using $v^2 = u^2 + 2as$ we get $0 = u^2 - 20g$ so $u = \sqrt{20g} = 14 = 20 \times \sin\alpha$ ✓
   $\sin\alpha = \frac{14}{20}$ giving $\alpha = 44.4^0$ (3sf) ✓
   (b) No acceleration horizontally, so distance $x = ut$ so $30 = 20t \times \cos 44.4$ giving $t = 2.1$ ✓
   Using $s = ut + \frac{1}{2}at^2$ vertically we get $y = 14 \times 2.1 - 4.9 \times 2.1^2 = 7.8$ m (2sf) ✓
   (6 marks)

4.
   (a) For a given point (x, y), horizontally, $s = x, u = 28 \times \cos\alpha, v = ?, a = 0, t = T$ ✓
   Vertically, $s = y, u = 28 \times \sin\alpha, v = ?, a = -g, t = T$ ✓
   Using $s = ut + \frac{1}{2}at^2$ horizontally we get $x = 28T\cos\alpha$ so $T = \frac{x}{28\cos\alpha}$ (I) ✓
   Using $s = ut + \frac{1}{2}at^2$ vertically we get $y = 28T\sin\alpha - \frac{1}{2}gT^2$ sub in (I) ✓
   So $y = 28\frac{x}{28\cos\alpha}\sin\alpha - \frac{1}{2}g\left(\frac{x}{28\cos\alpha}\right)^2 = x\tan\alpha - \frac{4.9 \times x^2}{(28\cos\alpha)^2} = x\tan\alpha - \frac{x^2}{160(\cos\alpha)^2}$ ✓
   So $k = 160$ ✓

   (b) Substitute $\alpha = 30, y = 2$ into the formula from part (a) to give ✓
   $$2 = x\tan 30 - \frac{x^2}{160\cos^2 30} = \frac{\sqrt{3}}{3}x - \frac{1}{120}x^2$$ ✓
   Solving the quadratic gives two values; $x = 3.66$ m ✓ or $x = 65.6$ m ✓ (both 3sf **(10 marks)**

5. (a)	After 4 seconds from *O*, horizontal speed $= u\cos\theta$	B1
	Vertical component of speed at *A* = $u + at$	M1
	$= u\sin\theta - 4g$	A1
	At *A*, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1
	$u\cos\theta = 15\cos 20$ $\quad$ $u\sin\theta = 15\sin 20 + 4g$	dM1
	$\theta = 72.4$ (72)	A1
	$u = 46.5$ (47)	A1 (7)

(b)	$-15\sin 20 = 15\sin 20 - gt$ or $0 = 15\sin 20t - \frac{1}{2}gt^2$	M1
	$t = 1.05$ (s) or 1.0 (s)	A1
		[2]
(c)	Total time = $4 + (1.05) + 4$	B1ft
	Range = $46.5 \times \cos 72.4 \times (8 + 1.05)$ (or $15\cos 20 \times 9.05$)	M1
	$= 128$ (m) or 127 (m) (130)	A1 (3)
		(12 marks)

6.(a)	Horizontal motion: $x = 3t$	B1
	Vertical motion: $y = 4t - \frac{g}{2}t^2$	M1, A1
	$\left(y = 4 \times \frac{x}{3} - \frac{g}{2} \times \frac{x^2}{9}\right)$, $\lambda = -\left(\frac{4\lambda}{3} - \frac{g\lambda^2}{18}\right)$	M1
	, $\frac{7\lambda}{3} = \frac{g\lambda^2}{18}$	M1
	$\lambda = \frac{42}{g}$ or 4.3 (4.29)	A1 (6)
(b)	At A: $v \rightarrow 3$ (m s^{-1})	B1
	$v\uparrow$ $4 - g \times \frac{14}{g}$	M1 A1
	$= -10$ (m s^{-1})	
	Speed $= \sqrt{(\text{their } 10)^2 + (3)^2}$	DM1
	$= \sqrt{109}$ (m s^{-1})	A1
	$\tan^{-1}\left(\frac{\text{their } 10}{3}\right)$ or $\tan^{-1}\left(\frac{3}{\text{their } 10}\right)$	DM1
	Direction $= 73.3°$ below the horizontal	A1 (7)
		[13 marks]

TOTAL 50 Marks