Mechanics Homework 13 Solutions

Section 1

(1)

(a)	$(\downarrow)$	$u_y = 25\sin 30° \ (=12.5)$		B1
		$12 = 12.5t + 4.9t^2$	−1 each error	M1 A2, 1, 0
		Leading to $t = 0.743$, 0.74		A1 (5)
(b)	$(\rightarrow)$	$u_x = 25\cos 30° \left(= \frac{25\sqrt{3}}{2} \approx 21.65\right)$		B1
		$OB = 25\cos 30° \times t \ (\approx 16.094\,58)$	ft their (a)	M1 A1ft
		$TB \approx 1.1$ (m)	awrt 1.09	A1 (4)
(c)	$(\rightarrow)$	$15 = u_x \times t \Rightarrow t = \frac{15}{u_x} (= \frac{2\sqrt{3}}{5} \approx 0.693$ or $0.69)$		M1 A1
	either $(\downarrow)$	$v_y = 12.5 + 9.8t \ (\approx 19.2896)$		M1
		$V^2 = u_x{}^2 + v_y{}^2 \ (\approx 840.840)$		
		$V \approx 29 \ (\text{ms}^{-1})$, 29.0		M1 A1 (5)
				(14 marks)

(2)

$$8.45 \times 1.15 = \frac{194.35}{n}$$

$$\Rightarrow n = \frac{194.35}{8.45 \times 1.15} = 20$$

so the size of Zain's sample is 20

[Here it has been used that 1knot = 1.15 mph]

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Section 2

(1) $\underline{u} = 5\hat{i} - \hat{j}$ $\quad \underline{a} = -\hat{i} + 2\hat{j}$ $\quad t = 3$ ✓

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$= 3(5\hat{i} - \hat{j}) + 4.5(-\hat{i} + 2\hat{j}) = 10.5\hat{i} + 6\hat{j}$ ✓

Initial position vector $= (3\hat{i} - 2\hat{j}) \Rightarrow$ final position vector

$= (3\hat{i} - 2\hat{j}) + (10.5\hat{i} + 6\hat{j}) = \underline{13.5\hat{i} + 4\hat{j}}$ ✓ 5

(2) $\underline{a} = 2\hat{i} + \hat{j}$ $\quad \underline{u} = -\hat{i} + 2\hat{j}$ $\quad t = 5$ ✓

$\underline{v} = \underline{u} + \underline{a}t = (-\hat{i} + 2\hat{j}) + 5(2\hat{i} + \hat{j}) = 9\hat{i} + 7\hat{j}$ ✓

Speed $= \sqrt{9^2 + 7^2} = \sqrt{130}$ ✓ $= \underline{11.4\,ms^{-1}}$ to 3sf

Direction $= \tan^{-1}(7/9) = \underline{37.9°}$ ✓ (anti-clockwise from +ve x-axis) 6

(3) $\underline{u} = 3\lambda\hat{i} + \lambda\hat{j}$ $\quad |\underline{u}| = 2\sqrt{10} \Rightarrow \sqrt{(3\lambda)^2 + \lambda^2} = 2\sqrt{10}$ ✓

$\Rightarrow 9\lambda^2 + \lambda^2 = 40$

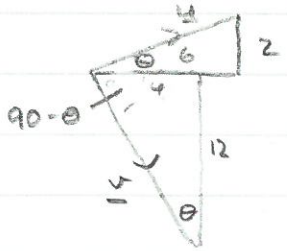
$10\lambda^2 = 40 \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = 2$ since $\lambda > 0$

$\underline{u} = 6\hat{i} + 2\hat{j}$ ✓

$\underline{a} = -\hat{i} - 7\hat{j}$

$t = 2$

$\underline{v} = \underline{u} + \underline{a}t = (6\hat{i} + 2\hat{j}) + 2(-\hat{i} - 7\hat{j}) = 4\hat{i} - 12\hat{j}$ ✓



similar triangles ∴ angle between $\underline{u}$ and $\underline{v} = \theta + (90° - \theta)$

$= 90° \Rightarrow$ perpendicular ✓

$|\underline{v}| = \sqrt{4^2 + 12^2} = 4\sqrt{10} = 2|\underline{u}| \Rightarrow$ speed doubled ✓

6

(4) A : $\underline{u} = (2\hat{i} + \hat{j})$ $\underline{a} = (\hat{i} - 2\hat{j})$ $t = 6$

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2 = 6(2\hat{i} + \hat{j}) + 18(\hat{i} - 2\hat{j}) = 30\hat{i} - 30\hat{j}$ ✓ ✓

Initial position = $3\hat{j}$ ⇒ position vector after 6 sec = $30\hat{i} - 27\hat{j}$ ✓

B : $\underline{u} = (3\hat{i} - \hat{j})$ $\underline{a} = 2\hat{i}$ $t = 6$

$s = \underline{u}t + \frac{1}{2}\underline{a}t^2 = 6(3\hat{i} - \hat{j}) + 18(2\hat{i}) = 54\hat{i} - 6\hat{j}$ ✓ ✓

Initially $\overrightarrow{AB} = 2\hat{i} - 2\hat{j}$ ⇒ initial position vector of B is $3\hat{j} + (2\hat{i} - \hat{j})$
$= 2\hat{i} + \hat{j}$

⇒ position vector of B after 6 sec $= (54\hat{i} - 6\hat{j}) + (2\hat{i} + \hat{j})$
$= 56\hat{i} - 5\hat{j}$ ✓

Distance between A and B after 6 sec $= \sqrt{(56-30)^2 + (-5+27)^2}$ ✓
$= \sqrt{1160} = 34.1$ m ✓ 8

(5)

(a)	$2ut = 735$		M1 A1	
	$0 = 3ut - \frac{1}{2}gt^2$		M1 A1	
	eliminating t		**dep.** M1	
	$u = 24.5$ *		A1	(6)
(b)	$t = \frac{735}{49} = 15$		M1 A1	(2)
(c)	Initially: $v^2 = (2u)^2 + (3u)^2$		M1	
	(7803.25)			
	$\frac{1}{2}mv^2 - \frac{1}{2}m\,65^2 = mgh$		M1 A1	
	$h = 180$ m (183 m)		A1	(4)
	OR $v_y^2 = 65^2 - (2u)^2$	(1824)	M1	
	$v_y^2 = (3u)^2 - 2gh$		M1 A1	
	$h = 180$ m (183 m)		A1	(4)
				(12 marks)

6.	(a) $\rightarrow \quad 30 = 2ut$	B1
	$\uparrow \quad -47.5 = 5ut - 4.9t^2$	M1 A1
	$-47.5 = 75 - 4.9t^2$ eliminating u or t	DM1
	$t^2 = \dfrac{75 + 47.5}{4.9} \left(= 25\right)$	DM1
	$t = 5$ * cso	A1 **(6)**
	(b) $30 = 2ut \Rightarrow 30 = 10u \Rightarrow u = 3$	M1 A1 **(2)**
	(c) $\uparrow \quad v = 5u - 9.8t = -34$ M1 requires both v and v	M1 A1
	$\rightarrow \quad v = 2u = 6$	A1
	$v^2 = 6^2 + (-34)^2$	DM1
	$v \approx 34.5 \ \left(\text{m s}^{-1}\right)$ accept 35	A1 **(5)**
		[13]
	Alternative to (c)	
	$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m \times g \times 47.5$ with $v_A^2 = 6^2 + 15^2 = 261$	M1 A(2,1,0)
	$v_B^2 = 261 + 2 \times 9.8 \times 47.5 \ \left(= 1192\right)$	DM1
	$v_B \approx 34.5 \ \left(\text{m s}^{-1}\right)$ accept 35	A1 **(5)**
	BEWARE : Watch out for incorrect use of $v^2 = u^2 + 2as$	