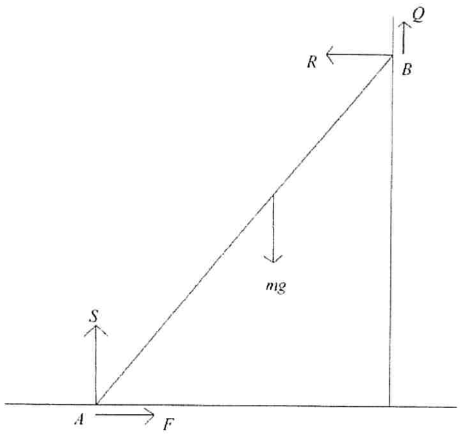
Mechanics Homework 14 Solutions

Section 1

Question Number	Scheme	Marks	
1			
(a)	$R = F$	B1	
	$S + Q = mg$	B1	
	$Q = \frac{2}{3}R, \quad F = \frac{1}{4}S$	B1	
	$Q = \frac{2}{3}R = \frac{2}{3}\times\frac{1}{4}S, \qquad S + \frac{1}{6}S = mg, \; S = \frac{6}{7}mg$	M1 A1	(5)
(b)	M(A) $\; mg \times x\cos 60 = Q \times 2l\cos 60 + R \times 2l\sin 60$	M1	
	M (B) $\; mg(2l - x)\cos 60 + F \times 2l\sin 60 = S \times 2l\cos 60$	A2	
	M (c of m) $\; Sx\cos 60 = Fx\sin 60 + R(2l - x)\sin 60 + Q(2l - x)\cos 60$		
	$mgx\cos 60 = \frac{1}{6}\times\frac{6}{7}mg \times 2l\cos 60 + \frac{1}{4}\times\frac{6}{7}mg \times 2l\sin 60$	DM1	
	$\frac{1}{2}x = \frac{1}{7}\times 2l \times \frac{1}{2} + \frac{3}{14}\times l\sqrt{3}$		
	$AG = x = 1.028.....l \;\; x = 1.03l$	A1	(5)
			[10]

2 (a)	Using $s = ut + \frac{1}{2}at^2$ Method must be clear $\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$ Answer given	M1 A1 A1	(3)
(b)	$\mathbf{j}$ component $= 0$: $10 + 5t - 4.9t^2$ quadratic formula: $t = \frac{5 \pm \sqrt{25+196}}{9.8} = \frac{5 \pm \sqrt{221}}{9.8}$ $T = 2.03$(s), 2.0 (s) positive solution only.	M1 DM1 A1	(3)
(c)	Differentiating the position vector (or working from first principles) $\mathbf{v} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$ (ms^{-1})	M1 A1	(2)
(d)	At B the $\mathbf{j}$ component of the velocity is the negative of the $\mathbf{i}$ component: $5 - 9.8t = -3$, $8 = 9.8t$, $t = 0.82$	M1 A1	(2)
(e)	$\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, speed $= \sqrt{3^2 + 3^2} = \sqrt{18} = 4.24$ (m s^{-1})	M1A1	(2) **[12]**

Question	Scheme
3	
(a/i)	The large data set contains data for the months May-October and there are 184 days between (1^{st}) May and (31^{st}) October.
(a/ii)	e.g. The large data set contains gaps
(b)	(Starting from 1^{st} May), each day the total amount of rainfall in Leuchars in 2015 decreases by 0.0027 mm
(c)	$x = 3 \Rightarrow T = 16.551 - 0.0027(3) = \underline{16.5429}$
(d)	*Idea that* The daily mean rainfall in Leuchars (in 2015) does not decrease at a steady rate, but fluctuates **IGNORE** references to 'extrapolation' – the question asks for discussion about the unreliability for any day in Leuchars in 2015, not just those outside of the data range

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Section 2

(1) a) $2\times3^3 - 4\times3^2 + 3 = \underline{21\text{m}}$ ✓ 1

b) $v = ds/dt$ ✓ $= 6t^2$ ✓ $- 8t$ ✓ 3

c) $6\times3^2 - 8\times3 = \underline{30\text{ms}^{-1}}$ ✓ 1

(2) a) $v = ds/dt$ ✓ $= 2e^{2t}$ ✓

$= 2e^{0.8} \approx \underline{4.45\text{ms}^{-1}}$ ✓ at $t = 0.4$. 3

b.) $a = dv/dt$ ✓ $= 4e^{2t}$ ✓

$= 4e^{0.6} \approx \underline{7.29\text{ms}^{-2}}$ ✓ at $t = 0.3$ 3

(3) a) $v = ds/dt$ ✓ $= \frac{\pi}{6}\cos\left(\frac{\pi t}{6}\right)$ ✓

$= \frac{\pi\sqrt{3}}{12} \approx \underline{0.453\text{ms}^{-1}}$ ✓ at $t = 1$ 3

b) $a = dv/dt$ ✓ $= -\frac{\pi^2}{36}\sin\left(\frac{\pi t}{6}\right)$ ✓

$= \frac{-\pi^2\sqrt{3}}{72} \approx \underline{-0.237\text{ms}^{-2}}$ ✓ at $t = 2$ ✓ 3

(4) a) $v = 0 \Rightarrow 2t^2 - 14t + 20 = 0$

$\Rightarrow t^2 - 7t + 10 = 0$ ✓

$\Rightarrow (t-2)(t-5) = 0 \Rightarrow t = 2$ or 5 ✓ 2

b.) $dv/dt = 0 \Rightarrow 4t - 14 = 0 \Rightarrow t = 3.5$ ✓

$v = 2\times3.5^2 - 14\times3.5 + 20 = -4.5\text{ms}^{-1}$ ✓ (min value of v)

$t = 0 \Rightarrow v = 20$

$t = 4 \Rightarrow v = -4$ ✓ so max. speed $= \underline{20\text{ms}^{-1}}$ ✓ 4

c.) Displacement $s = \int v\,dt = \frac{2t^3}{3} - 7t^2 + 20t + c$ ✓

Since initially at origin, $s = 0$ when $t = 0 \Rightarrow c = 0$

At $t = 2$, $s = 17\frac{1}{3}$ m ✓ At $t = 4$, $s = 10\frac{2}{3}$ ✓

$\Rightarrow$ Total distance $= 17\frac{1}{3} + (17\frac{1}{3} - 10\frac{2}{3}) = \underline{24\text{m}}$ ✓ 4

⑤ a) $v = \int a\,dt = \frac{1}{2} \times \frac{1}{3/2}(2t+1)^{3/2} + c = \frac{1}{3}(2t+1)^{3/2} + c$ ✓

$v = 2$ when $t = 0 \Rightarrow 2 = 1/3 + c \Rightarrow c = 1\,2/3$ ✓

$v = \frac{1}{3}(2t+1)^{3/2} + 1\,2/3 = \underline{10\tfrac{2}{3}\,ms^{-1}}$ ✓ at $t = 4$ 3

b) $s = \int v\,dt$ ✓ $= \frac{1}{6} \times \frac{1}{5/2}(2t+1)^{5/2} + 1\,2/3\,t + k$

$= \frac{1}{15}(2t+1)^{5/2} + 1\,2/3\,t + k$ ✓

$s = 4$ when $t = 0 \Rightarrow 4 = \frac{1}{15} + k \Rightarrow k = 3\,14/15$ ✓

$s = \frac{1}{15}(2t+1)^{5/2} + 1\,2/3\,t + 3\,14/15 \approx \underline{232\,m}$ ✓ 4

⑥ a) i) $s = \int v\,dt = 0.1 \times \frac{1}{5/2} t^{5/2} + c = 0.04t^{5/2} + c$ ✓

$s = 0$ when $t = 0 \Rightarrow c = 0$ ✓

$s = 0.04t^{5/2} = \underline{2.24\,ms^{-1}}$ ✓ when $t = 5$ 3

ii) $s = 0.04 \times 20^{5/2}$ ✓ $= \underline{71.6\,m}$ ✓✓ when $t = 20$. 3

iii) After $t = 20$, $s = \int 0.1(40-t)^{3/2}\,dt$ ✓ $= -0.04(40-t)^{5/2} + k$ ✓

$t = 20 \Rightarrow s = 71.6 \Rightarrow 71.6 = -71.6 + k \Rightarrow k = 143.1$ ✓

$t = 32 \Rightarrow s = -0.04 \times (40-32)^{5/2} + 143.1 = \underline{136\,m}$ ✓

3sf. 4

b) Velocity would be negative beyond 40 sec. ✓ 1