Mechanics Homework 15 Solutions

Section 1

Q		Solution	Marks	
1	(a)	Inextensible string	B1	(1)
	(b)	$4mg - T = 4ma$	M1A1	
		$T - 2mg\sin\alpha - F = 2ma$	M1A1	(4)
	(c)	$F = 0.25R$	B1	
		$R = 2mg\cos\alpha$	B1	
		$\cos\alpha = 0.8$ or $\sin\alpha = 0.6$	B1	
		Eliminating R, F and T	M1	
		$a = 0.4g = 3.92$	A1	(5)
	(d)	$v^2 = 2 \text{ x } 0.4gh$	M1	
		$-2mg\sin\alpha - F = 2ma'$	M1	
		$a' = -0.8g$	A1	
		$0^2 = 0.8gh - 2\text{x } 0.8g \text{ x } s$	M1	
		$s = 0.5h$	A1	
		$XY = 0.5h + h = 1.5h$	A1	(6)
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2	a)	$\frac{dv}{dt} = t - 4$		
		$v = \frac{1}{2}t^2 - 4t\,(+c)$	M1 A1	
		$t = 0 \quad v = 6 \qquad \Rightarrow c = 6$	M1	
		$\therefore v = \frac{1}{2}t^2 - 4t + 6$	A1	(4)
	(b)	$v = 0 \quad 0 = t^2 - 8t + 12$	M1	
		$(t-6)(t-2) = 0$	DM1	
		$t = 6 \quad t = 2$	A1	(3)
	(c)	$x = \frac{t^3}{6} - 2t^2 + 6t + k$	M1 A1 ft	
		$x_6 - x_2 = \frac{6^3}{6} - 2\times 6^2 + 6^2 + k - \left(\frac{2^3}{6} - 2\times 2^2 + 6\times 2 + k\right) = -5\frac{1}{3}$	DM1	
		$\therefore$ Distance is $5\frac{1}{3}$ m	A1	(4)
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① $\underline{r} = (3t-2)\hat{\underline{i}} + (4t-2t^2)\hat{\underline{j}}$

a) $\underline{v} = \dot{\underline{r}} = 3\hat{\underline{i}} + (4-4t)\hat{\underline{j}}$

$\underline{a} = \ddot{\underline{r}} = -4\hat{\underline{j}}$ 4

b.) At $t=0$, $\underline{v} = 3\hat{\underline{i}} + 4\hat{\underline{j}} \Rightarrow$ speed $= \sqrt{3^2+4^2} = \underline{5\,ms^{-1}}$ 2

c) Moving parallel to x-axis $\Rightarrow$ $\hat{\underline{j}}$-component of $\underline{v}$ is 0.

$\Rightarrow 4-4t=0 \Rightarrow \underline{t = 1\,sec}$ 2

d) Stationary $\Rightarrow \underline{v} = 0$. This is never the case as $\hat{\underline{i}}$-component $= 3$ at all times. 2

② $\underline{r} = (2t-1+\cos t)\hat{\underline{i}} + (\sin 2t)\hat{\underline{j}}$

a.) Touches x-axis when $\hat{\underline{j}}$-component $=0 \Rightarrow \sin 2t = 0$. $\Rightarrow t = \underline{\pi/2}$ 2

b) $\underline{v} = (2-\sin t)\hat{\underline{i}} + (2\cos 2t)\hat{\underline{j}} = \hat{\underline{i}} - 2\hat{\underline{j}}$ at $t = \pi/2$

$\underline{a} = -\cos t\,\hat{\underline{i}} - 4\sin 2t\,\hat{\underline{j}} = \underline{0}$ at $t = \pi/2$ 4

③ $\underline{F} = 2000t\,\hat{\underline{i}} - 4000\hat{\underline{j}}$ $\quad \underline{F} = m\underline{a} \Rightarrow \underline{a} = 4t\,\hat{\underline{i}} - 8\hat{\underline{j}}$

a.) $\underline{v} = \int \underline{a}\,dt = (2t^2 + c_1)\hat{\underline{i}} + (-8t + c_2)\hat{\underline{j}}$

$t=0$, $\underline{v} = 10\hat{\underline{i}} \Rightarrow c_1 = 10, c_2 = 0$

$\underline{v} = (2t^2+10)\hat{\underline{i}} - 8t\,\hat{\underline{j}} = 18\hat{\underline{i}} - 16\hat{\underline{j}}$ at $t=2$

$\Rightarrow$ speed $= |\underline{v}| = \sqrt{18^2+16^2} = \underline{24.1\,ms^{-1}}$ to 3sf 5

b) $\underline{r} = \int \underline{v}\,dt = \left(\frac{2}{3}t^3 + 10t + c_3\right)\hat{\underline{i}} + (-4t^2 + c_4)\hat{\underline{j}}$

$\underline{r} = \underline{0}$ at $t=0 \Rightarrow c_3, c_4 = 0$

$\Rightarrow \underline{r} = \left(\frac{2}{3}t^3 + 10t\right)\hat{\underline{i}} - 4t^2\hat{\underline{j}} = \frac{76}{3}\hat{\underline{i}} - 16\hat{\underline{j}}$

Distance $= |\underline{r}| = \sqrt{\left(\frac{76}{3}\right)^2 + 16^2} = \underline{30.0\,m}$ 4

④ $\underline{r} = 10e^{-t}\hat{\underline{i}} + 2\hat{\underline{j}}$

a) $\underline{a} = \dot{\underline{v}} = -10e^{-t}\hat{\underline{i}} = \frac{-10}{e} \approx -3.68\ ms^{-2}$ ✓ 3

b.) $\underline{r} = \int \underline{v}\,dt = (-10e^{-t} + c_1)\hat{\underline{i}} + (2t + c_2)\hat{\underline{j}}$ ✓

$t=0,\ \underline{r} = 2\hat{\underline{i}} + 3\hat{\underline{j}} \Rightarrow c_1 = 12,\ c_2 = 3$

$\underline{r} = (-10e^{-t} + 12)\hat{\underline{i}} + (2t+3)\hat{\underline{j}} = (12 - {}^{10}/_{e})\hat{\underline{i}} + 5\hat{\underline{j}}$ at $t=1$ ✓ 4

$= 8.32$ to 3sf

⑤ $\underline{a} = -4\cos 2t\,\hat{\underline{i}} - 4\sin 2t\,\hat{\underline{j}}$

a.) $\underline{v} = \int \underline{a}\,dt = -2\sin 2t\,\hat{\underline{i}} + 2\cos 2t\,\hat{\underline{j}}$ ✓

speed $= |\underline{v}| = \sqrt{4\sin^2 2t + 4\cos^2 2t} = \sqrt{4(\sin^2 2t + \cos^2 2t)} = \underline{\underline{2\ ms^{-1}}}$

$\Rightarrow$ constant ✓ 3

b.) $\underline{r} = \int \underline{v}\,dt = \cos 2t\,\hat{\underline{i}} + \sin 2t\,\hat{\underline{j}}$ ✓

Distance from origin : $|\underline{r}| = \cos^2 2t + \sin^2 2t = 1m$ ✓

Path is a circle, radius 1m, centre (0,0). ✓ 4

⑥ For P, $\underline{a} = \lambda\underline{i} + \lambda\underline{j}$ $\quad |\underline{a}| = t = \sqrt{\lambda^2 + \lambda^2} = t = \lambda\sqrt{2} = t$

$\lambda = \frac{\sqrt{2}}{2}t$ ✓

For Q, $\underline{a} = \lambda\hat{\underline{i}} - \lambda\underline{j}$ ✓

For P, $\underline{v} = \int \underline{a}\,dt = \left(\frac{\sqrt{2}}{4}t^2 + c_1\right)\hat{\underline{i}} + \left(\frac{\sqrt{2}}{4}t^2 + c_2\right)\hat{\underline{j}}$

$t=0,\ \underline{v}_p = 2\hat{\underline{i}} - 5\hat{\underline{j}} \Rightarrow c_1 = 2,\ c_2 = -5$ $\quad \underline{v}_P = \left(\frac{\sqrt{2}}{4}t^2 + 2\right)\underline{i} + \left(\frac{\sqrt{2}}{4}t^2 - 5\right)\underline{j}$ ✓

Similarly $\underline{v}_Q = \left(\frac{\sqrt{2}}{4}t^2\right)\underline{i} + \left(2 - \frac{\sqrt{2}}{4}t^2\right)\hat{\underline{j}}$ ✓

$|\underline{v}_P| = |\underline{v}_Q|$ when $\left(\frac{\sqrt{2}}{4}t^2 + 2\right)^2 + \left(\frac{\sqrt{2}}{4}t^2 - 5\right)^2 = \left(\frac{\sqrt{2}}{4}t^2\right)^2 + \left(2 - \frac{\sqrt{2}}{4}t^2\right)^2$ ✓

$\Rightarrow \frac{1}{4}t^4 - \frac{3}{2}\sqrt{2}t^2 + 29 = \frac{1}{4}t^4 - \sqrt{2}t^2 + 4 \Rightarrow 25 = \frac{1}{2}\sqrt{2}t^2$ ✓ 6

$\Rightarrow t = 5.95$