

## Pure 7 – Problems on Circles

Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

### Section 1 – Review of previous topics. Please **complete** all questions.

- Express each of the following in the form  $a\sqrt{2} + b\sqrt{3}$ , where  $a$  and  $b$  are integers.
  - $\sqrt{27} + 2\sqrt{50}$
  - $\sqrt{6}(\sqrt{3} - \sqrt{8})$
- Given that  $6^{y+1} = 36^{x-2}$ , a) express  $y$  in the form  $ax + b$ , b) find the value of  $4^{x - \frac{1}{2}y}$ .
- The circle  $C$  has the equation  $x^2 + y^2 - 12x + 8y + 16 = 0$ .
  - Find the coordinates of the centre of  $C$  and the radius of  $C$
  - Given that  $C$  crosses the  $x$ -axis at the points  $A$  and  $B$ , find the length  $AB$ , giving your answer in the form  $k\sqrt{5}$
- Solve the equation  $x^2 + 10x + 13 = 0$ , giving your answers in the form  $a + b\sqrt{3}$ , where  $a$  and  $b$  are integers.
  - Hence find the set of values of  $x$  for which  $x^2 + 10x + 13 > 0$

### Section 2 – Consolidation of this week's topic. Please **complete** all questions.

- Prove that the points  $A(-10, -12)$ ,  $B(6, 18)$ , and  $C(-2, -14)$  lie on a semi-circle.
 

**(4 marks)**
- Write the equation of the tangent to the circle with centre  $(4, -3)$  at the point  $P(-2, -1)$ 

**(4 marks)**
- On a map, three villages are situated at points  $A(2, -5)$ ,  $B(10, 1)$ , and  $C(9, -6)$  and all lie on the circumference of a circle
  - Find the equations of the perpendicular bisectors of  $AB$  and  $AC$
  - Hence work out the centre and equation of the circle and show that the triangle formed by the villages is right angled
 

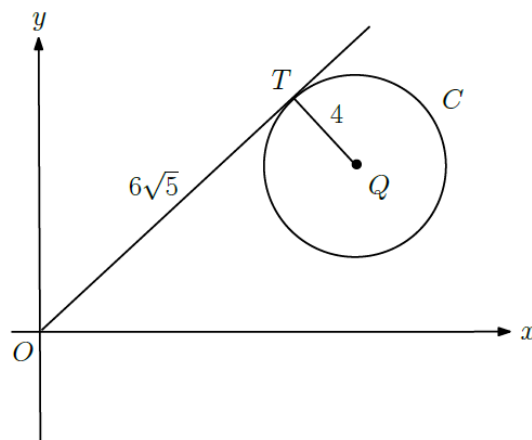
**(10 Marks)**
- The equation of a circle, centre  $C$ , is  $x^2 + y^2 - 4x - 12y + 15 = 0$ 
  - Prove that the circle does not intersect the  $x$ -axis
  - $P$  is the point  $(8, 1)$ . Find the length  $CP$  and determine whether  $P$  lies inside or outside the circle
  - Write the set of values of  $k$  for which  $3y - 4x = k$  is a tangent to the circle
 

**(15 Marks)**

**(Total 38 Marks)**

Section 3 – Extension question. If you are aiming for a top grade, you should attempt these questions.

1. A circle, centre  $C$ , has equation  $x^2 + y^2 - 20x + 10y + 25 = 0$  and meets the  $y$  – axis at  $Q$ . The tangent at  $P(16, 3)$  meets the  $y$  – axis at  $R$ . Work out the area of the triangle  $PQR$ .
2. Find the equation of the circle passing through the points  $A(1,5)$ ,  $B(2,3)$ ,  $C(-2, 1)$
3. The figure below shows a circle  $C$  with centre  $Q$  and radius 4 and the point  $T$  which lies on  $C$ . The tangent to  $C$  at the point  $T$  passes through the origin  $O$  and  $OT = 6\sqrt{5}$ .



- a) Given that the coordinates of  $Q$  are  $(11, k)$ , where  $k$  is a positive constant, find the exact value of  $k$ ,
- (b) Find an equation for  $C$ .