

Pure 8 – Algebraic Division and the Factor Theorem

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

- Find the centre and radius of each of these circles:
 - $x^2 + 4x + y^2 + 6y - 17 = 0$
 - $x^2 + y^2 - 6x - 8y = 0$
- Find the equation of the tangent to the circle whose equation is $(x + 5)^2 + (y - 1)^2 = 65$ at the point $(3, 2)$
- Find the equation of the line passing through the point $A(4, -2)$ and perpendicular to the line l whose equation is $2x - y - 5 = 0$.
 - Find the coordinates of the foot of the perpendicular from A to the line l .
 - Hence find the perpendicular distance from A to l .
- Find p , q , and r such that $5x^2 - 2x + 1 \equiv p(x - q)^2 + r$. Hence, find the minimum value of $5x^2 - 2x + 1$ and the value of x for which it occurs.

Section 2 – Consolidation of this week's topic. Please complete all questions.

- Divide

- | | |
|-----------------------------------------------------|---------------------------------------------|
| (a) $x^2 - x - 90$ by $(x + 9)$ | (b) $3x^2 - 19x - 14$ by $(x - 7)$ |
| (c) $8x^2 + 14x - 15$ by $(2x + 5)$ | (d) $x^3 - 2x + 1$ by $(x - 1)$ |
| (e) $x^3 - 10x^2 - 10x - 11$ by $(x - 11)$ | (f) $6x^3 - 13x^2 - 19x + 12$ by $(3x + 4)$ |
| (g) $6x^4 - 19x^3 + 23x^2 - 26x + 21$ by $(2x - 3)$ | |
| (h) $10x^4 + 33x^3 - 57x^2 + 5x + 1$ by $(5x - 1)$ | |

(24 marks)

- Using the factor theorem, show that:

- $(x + 6)$ is a factor of $x^3 + 4x^2 - 9x + 18$
- $(x - 8)$ is a factor of $2x^3 - 13x^2 - 20x - 32$
- $(3x - 1)$ is a factor of $3x^3 + 11x^2 - 25x + 7$
- $(5x + 2)$ is a factor of $10x^3 + 19x^2 - 39x - 18$

(12 marks)

3. Fully Factorise:

- (a) $x^3 + 3x^2 - 16x + 12$ (b) $x^3 - 6x^2 - 55x + 252$ (c) $6x^3 + 19x^2 + x - 6$
(d) $x^4 - 13x^2 - 48$

(16 marks)

4.

- (a) $2x^4 + px^3 - 6x^2 + qx + 6$ is divisible by $(x - 1)$. Use this information to write an equation in p and q .
(b) $2x^4 + px^3 - 6x^2 + qx + 6$ is divisible by $(x + 3)$. Use this information to write an equation in p and q .
(c) Find the values of p and q

(8 marks)

5.

- (a) Work out the value of a when $2x^3 + ax^2 - 4x + 1$ is divisible by $(x - 2)$.
(b) Work out the value of b when $x^4 + (b^2 + 1)x^3 + bx^2 + 7x - 15$ is divisible by both $(x + 5)$ and $(x - 1)$.
(c) Work out the values of p and q when $2x^4 + px^3 - 6x^2 + qx + 6$ is divisible by $x^2 + 2x - 3$

(12 marks)

6. Do the equations: $x^3 - 4x^2 - 31x + 70$, $x^2 + 3x - 10$, and $x^2 - 9x + 14$ have a common factor? If so what is it?

(4 marks)

7. Find the values of a and b if $(5x - 4)$ and $(x + 3)$ are factors of $ax^2 + 33x + b$

(4 marks)

8.

- (a) A circle's area is $(4\pi x^2 - 12\pi x + 9\pi)\text{m}^2$. Work out its radius.
(b) The volume of a square-based pyramid is $(2x^3 - 5x^2 - 24x + 63)\text{cm}^3$. The height is $(2x + 7)\text{cm}$. Work out the length of the side of the square base.

(9 marks)

9.

- (a) The velocity of a moving body is $(2t^3 - 19t^2 + 57t - 54)\text{ms}^{-1}$ at any time t . What are the times when the body is stationary?
(b) The acceleration of the same body is $(6t^2 - 38t + 57)\text{ms}^{-2}$. Work out
i. The acceleration of the body when the velocities are zero.
ii. The exact times when the acceleration is zero

(11 marks) (Total 100 Marks)

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

1. Find the LCM and HCF of $x^3 + 7x^2 - 53x - 315$ and $x^3 + 21x^2 + 143x + 315$
2. (from paper U)

$$f(x) \equiv x^3 + (a+2)x^2 - 2x + b,$$

where a and b are non zero constants.

It is given that $(x-2)$ and $(x+a)$ are factors of $f(x)$, $a > 0$.

- a) By forming two equations show that $a = 3$ and find the value of b . (6)
- b) Solve the equation $f(x) = 0$. (2)