

Pure 9 – Binomial Expansion

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

- Factorise fully: $2x^4 - x^3 - 5x^2 - 2x$.
Hence solve for x : $2x^4 - x^3 - 5x^2 - 2x = 0$
- Find the equations of the circles which pass through each of these sets of points.
 - $(1, 0), (7, -6), (7, 0)$
 - $(6, 4), (5, 9), (-10, 6)$
 - $(0, 0), (3, 7), (10, 0)$
- By considering the discriminant, or otherwise, find the range of values of k that give each of these equations two distinct real roots.
 - $x^2 + 3x + k = 0$
 - $3x^2 + kx + 2 = 0$
 - $k(x^2 + 1) = x - k$

Section 2 – Consolidation of this week's topic. Please complete all questions.

- Write down the values of :

(a) $5!$ (b) $7!$ (c) $11!$

(d) 5C_2 (e) 9C_3 (f) ${}^{11}C_7$ (g) ${}^{13}C_8$

(h) $\binom{5}{3}$ (i) $\binom{10}{1}$ (j) $\binom{13}{5}$ (k) $\binom{20}{6}$

(11 Marks)

- Find the first four terms of these binomial expansions in ascending powers of x

(a) $(1 + x)^8$ (b) $(1 - 3x)^7$ (c) $(1 + 2x)^9$

(d) $(2 - 3x)^6$ (e) $(x - 2)^8$ (f) $(2x - 1)^{10}$

(24 marks)

- Find the terms indicated in each of these expansions and simplify your answers

(a) $(p + 5)^5$ term in p^2 (b) $(4 + y)^9$ term in y^5 (c) $(z + \frac{3}{2})^8$ term in z^6

(d) $(2a - 3b)^{10}$ terms in a^5 and b^4

(10 Marks)

4. Fully expand and simplify each of these expressions

(a) $3x(2x - 5)^5$ (b) $(2 + x)^4(1 + x)$ (c) $(5 - 2x)^3 + (3 + 2x)^4$

(d) $(2 + \sqrt{3})^4 + (1 - \sqrt{3})^4$

(15 marks)

5.

(a) Expand $(1 + 4x)^6$ in ascending powers of x up to and including the term in x^2

(b) Use your answer to part (a) to estimate the value of $(1.04)^6$

(5 marks)

6.

(a) Expand $(1 - 2x)^7$ in ascending powers of x up to and including the term in x^3

(b) Use your answer to part (a) to estimate the value of $(0.99)^7$

(5 marks)

7. Use the binomial theorem to work out the value of:

(a) 1.015^5 correct to 4 decimal places

(b) $\left(\frac{199}{100}\right)^{10}$ correct to five significant figures

(6 marks)

8. The first three terms in the expansion of $(1 + ax)^n$ are $1 + 35x + 490x^2$. Given that n is a positive integer find the value of (a) n and (b) a .

(6 marks)

9. Given that $(1 + bx)^n \equiv 1 - 24x + 252x^2 \dots$ For a positive integer n find the value of (a) n and (b) b .

(6 marks)

10. In the expansion of $(1 + 2x)^n$, n is a positive integer. The coefficient of x^2 is eight times the coefficient of x . Find the value of n .

(6 marks)

11. In the expansion of $(1 + \frac{x}{2})^n$, n is a positive integer. The coefficients of x^4 and x^5 are equal. Calculate the value of n .

(6 marks)

(Total 100 Marks)

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

The binomial coefficients are given by

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad k \in \mathbb{N}, \quad n \in \mathbb{N} \cup \{0\}.$$

Show directly from the above definition that

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}. \quad (6)$$