Pure 10 – Binomial expansion

Please **complete** this homework by __________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop-in session.

Section 1 – Review of previous topics. Please complete all questions.

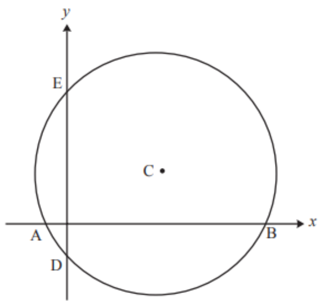
1. Simplify $\frac{1+\sqrt{10}}{\sqrt{10}-3}$.

2. Solve the equation $3x = \sqrt{5}(x+2)$ giving your answer in the form $a + b\sqrt{5}$ where a and b are rational.

3. Simplify $\frac{x^{\frac{3}{2}}-x}{x^{\frac{1}{2}}}$.

4. Simplify $\frac{x+1}{x^{\frac{1}{2}}+x^{-\frac{1}{2}}}$.

5.



The figure above shows a sketch of the circle with equation $x^2 + y^2 - 20x - 4y = 21$ and centre C. The points A, B, D and E are the intersections of the circle with the axes. Determine

i) the radius of the circle and the coordinates of C;
ii) verify that B is the point (21,0) and find the coordinates of A, D and E;
iii) find the equation of the perpendicular bisector of BE and verify that this line passes through C.

Section 2 – Consolidation of this week's topic. Please complete all questions.

1. Find the binomial expansion of each of the following in ascending powers of x up to and including the term x^3, for $|x| < 1$

   a) $2(1+x)^{-3}$ b) $\sqrt[3]{1-x}$ *(4 marks)*

2. Expand the following in ascending powers of x up to and including the term x^3, and state the set of values of x for which each expansion is valid.

   a) $(4+x)^{\frac{1}{2}}$ b) $(3-x)^{-3}$ *(6 marks)*

3. Find the coefficient of x^2 in the series expansion of $\frac{2+x}{\sqrt{4-2x}}$, $|x| < 2$ *(4 marks)*

4. The first three terms in the expansion of $(1+ax)^b$, in ascending powers of x, for $|ax| < 1$, are

   $$1 - 6x + 24x^2$$

   a) Find the values of the constants a and b. *(4 marks)*
   b) Find the coefficient of x^3 in the expansion *(2 marks)*

5. Find a counter-example to disprove each of the following statements, demonstrating why it is a counter-example:

   a) Every triangle has at least one angle greater than 60°
   b) $a < b \Rightarrow a^2 < b^2$ for all a,b
   c) (n! + 1) is a prime number for all positive integers n
   d) The equation of a straight line in 2 dimensions can always be written in the form y=mx+c for some m,c
   e) $k\pi$ is irrational for all non-zero values of k
   f) The graph of the function $y = ax^2 + bx + c$ is a parabola for all a,b,c
   g) $n^2 + n + 41$ is a prime number for all positive integers n

   (3 marks each)

6. Use the method of proof by exhaustion to prove the following statements;
   a) Discounting reflections and rotations, there are four distinct triangles with a perimeter of 11cm and every side having an integer length in cm.
   b) The product of any two consecutive positive integers ends in 0, 2, or 6. (Hint consider the final digits)

   (3 marks each)

7. Prove the following by direct deduction;
   a) $x^2 + 6x + 11$ is positive for all real values of x.
   b) If a,b,c,d are consecutive integers in ascending order, their sum is equal to $cd - ab$.
   c) The triangle whose vertices are (2,1), (5,2) and (4,5) is isosceles and right-angled.
   d) If the equation $x^2 + kx + 2k = 0$, where k is a positive constant, has two distinct real roots, then $k>8$.

(3 marks each)

(Total = 59 marks)