Pure 13 – Vectors 3

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Given that $q \neq 0$, find the value of the constant q such that the equation $x^2 + 2qx - 2q = 0$ has a repeated root.

2. Given that the x axis is a tangent to the curve with the equation $y = x^2 + rx - 2x + 4$, find the two possible values of the constant r.

3. Solve the equation $25^x = 5^{4x+1}$.

4. Show that $\frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}}$ can be written in the form $k\sqrt{7}$ where k is an integer to be found.

5. Given that the point with coordinates $\left(1 + \sqrt{3},\ 5\sqrt{3}\right)$ lies on the curve with the equation $y = 2x^2 + px + q$, find the values of the rational constants p and q.

6. Find the coordinates of the turning point of the curve with equation $y = 3 - 5x - 2x^2$. Sketch this curve showing the coordinates of any points of intersection with the coordinate axes.

1. Find the unit vector in the direction $2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}.$ (2)

2. Vectors **a** and **b** are defined by $\boldsymbol{a} = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}.$

   a. Find $\boldsymbol{a} - \boldsymbol{b}$.
   b. Find $2\boldsymbol{a} - 3\boldsymbol{b}$.
   c. State with a reason whether each of these vectors is parallel to $3\boldsymbol{i} + 3\boldsymbol{j} - 24\boldsymbol{k}.$ (6)

3. The position vector of the point A is $-3\boldsymbol{i} + 6\boldsymbol{j} + 4\boldsymbol{k}$ and $\overrightarrow{AB} = 7\boldsymbol{i} - 8\boldsymbol{j} - \boldsymbol{k}$ and the coordinates of point C are $(2, -2, -1)$. Find:

   a. the position vectors of B and C;
   b. $\overrightarrow{AC}$.

   Find the exact value of:

   c. the distance between A and C;
   d. $\left| \overrightarrow{OC} \right|$. (9)

4. The vectors a and b are defined by $\boldsymbol{a} = \begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}.$

   Given that $2\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} 3 \\ -6 \\ 10 \end{pmatrix}$ find the values of p, q and r. (4)

5. Given that $\boldsymbol{a} = 3t\boldsymbol{i} - 12t\boldsymbol{j} + 4t\boldsymbol{k}$ and $|\boldsymbol{a}| = 39$ find the possible values of t. (3)

6. Find the angles that the vector $\overrightarrow{AB} = -2\boldsymbol{i} + 5\boldsymbol{j} - 3\boldsymbol{k}$ makes with each of the positive coordinate axes to 1 d.p. (7)

7. The points A, B and C have coordinates $(4, 2, 7)$, $(5, 7, 6)$ and $(4, 12, 7)$ respectively.
    a) Show that triangle ABC is isosceles. *(3)*
    b) Find the area of triangle ABC. *(3)*

8. The points A and B have position vectors $-10\boldsymbol{i} + 10\boldsymbol{j} + 23\boldsymbol{k}$ and $22\boldsymbol{i} + p\boldsymbol{j} - 14\boldsymbol{k}$ respectively, relative to a fixed origin 0, where p is a constant. Given that triangle OAB is isosceles, find three possible positions of point B. *(9)*

9. The diagram shows a tetrahedron OABC
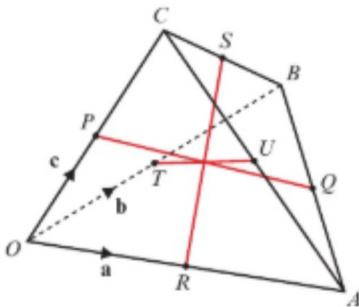
$\overrightarrow{OA} = \boldsymbol{a}$ $\overrightarrow{OB} = \boldsymbol{b}$ $\overrightarrow{OC} = \boldsymbol{c}$
P, Q, R, S, T, U are the midpoints of OC, AB, OA, BC, OB, AC respectively.

Prove that the line segments PQ, RS, TU meet at a point and bisect each other.

(9)

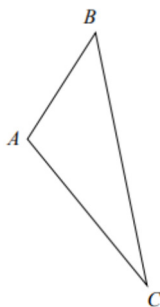10. The diagram shows a sketch of a triangle ABC.

Figure 2

Given that $\overrightarrow{AB} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ and $\overrightarrow{BC} = \boldsymbol{i} - 9\boldsymbol{j} + 3\boldsymbol{k}$ show that angle $BAC = 105.9°$ to one decimal place *(5).*

TOTAL: 60 marks