Pure 17 – Further Integration and Optimisation

Please **complete** this homework by __________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1 $\quad f(x) \equiv 2x^3 + 5x^2 - 1.$

a Find $f'(x)$.

b Find the set of values of x for which $f(x)$ is increasing.

2 $\quad$ The curve C has the equation $y = x^3 - x^2 + 2x - 4$.

a Find an equation of the tangent to C at the point $(1, -2)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

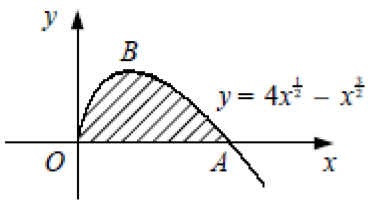
b Prove that the curve C has no stationary points.

3 $\quad$ Evaluate

a $\int_1^3 \left(2 - \frac{1}{x^2}\right)\, dx$ $\qquad$ **b** $\int_{-2}^{-1} \left(6x + \frac{4}{x^3}\right)\, dx$ $\qquad$ **c** $\int_1^4 \left(3x^{\frac{1}{2}} - 4\right)\, dx$

d $\int_{-1}^{2} \frac{4x^4 - x}{2x}\, dx$ $\qquad$ **e** $\int_1^8 \left(x - x^{-\frac{1}{3}}\right)\, dx$ $\qquad$ **f** $\int_2^3 \frac{1 - 6x^3}{3x^2}\, dx$

4



The diagram shows the curve with the equation $y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

The curve meets the x-axis at the origin, O, and at the point A.

a Find the coordinates of the point A. **(2)**

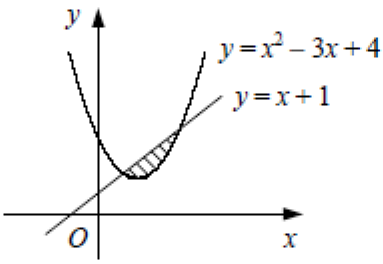
The curve has a maximum at the point B.

b Find the x-coordinate of the point B. **(5)**

c Find the area of the shaded region enclosed by the curve and the x-axis. **(4)**

Section 2 – Consolidation of this week's topic. Please <u>complete</u> all questions.

1



The diagram shows the curve $y = x^2 - 3x + 4$ and the straight line $y = x + 1$.

a Find the coordinates of the points where the curve and line intersect. (5)

b Find the area of the shaded region enclosed by the curve and the line.

2 In each part of this question sketch the given curve and line on the same set of coordinate axes and find the area of the region enclosed by the curve and line.

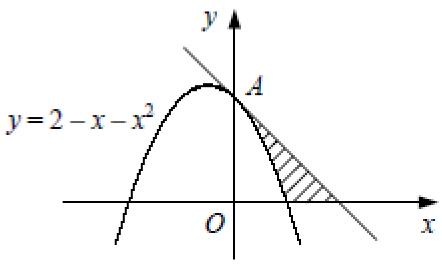
a $y = 9 - x^2$ and $y = 6 - 2x$ **b** $y = x^2 - 4x + 4$ and $y = 16$

c $y = x^2 - 5x - 6$ and $y = x - 11$ **d** $y = \sqrt{x}$ and $x - 2y = 0$

(24)

(6 marks each)

3



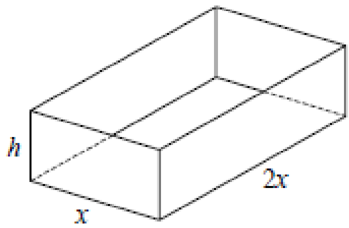
The diagram shows the curve with the equation $y = 2 - x - x^2$ and the tangent to the curve at the point A where it crosses the y-axis.

a Find an equation of the tangent to the curve at A. (4)

b Show that the area of the shaded region enclosed by the curve, the tangent to the curve at A and the x-axis is $\frac{5}{6}$. (7)

Please turn over

4



The diagram shows a baking tin in the shape of an open-topped cuboid made from thin metal sheet. The base of the tin measures x cm by $2x$ cm, the height of the tin is h cm and the volume of the tin is 4000 cm^3.

a Find an expression for h in terms of x.

b Show that the area of metal sheet used to make the tin, A cm^2, is given by

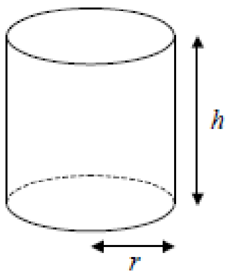
$$A = 2x^2 + \frac{12\,000}{x}.$$

c Use differentiation to find the value of x for which A is a minimum.

d Find the minimum value of A.

e Show that your value of A is a minimum. (10)

5



The diagram shows a closed plastic cylinder used for making compost. The radius of the base and the height of the cylinder are r cm and h cm respectively and the surface area of the cylinder is 30 000 cm^2.

a Show that the volume of the cylinder, V cm^3, is given by

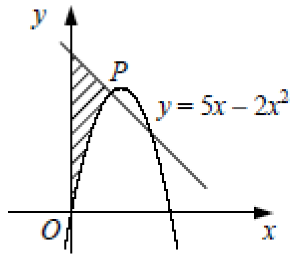
$$V = 15\,000r - \pi r^3.$$

b Find the maximum volume of the cylinder and show that your value is a maximum. (10)

Total: 60 Marks

Section 3 – Extension questions. If you are aiming for a top grade, you should attempt these questions.

1



1

The diagram shows the curve $y = 5x - 2x^2$ and the normal to the curve at the point P (1, 3).

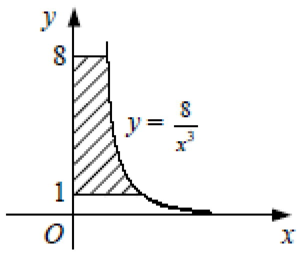
a Find an equation of the normal to the curve at P.

The shaded region is bounded by the curve, the normal to the curve at P and the y-axis.

b Show that the area of the shaded region is $\frac{5}{3}$.

2 **a** Evaluate $\int_1^2 \frac{8}{x^3}\,dx$.

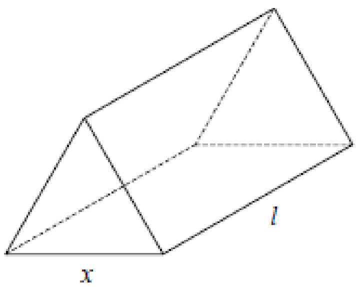
2



The diagram shows the curve with the equation $y = \frac{8}{x^3}$, $x > 0$.

b Using your answer to part **a**, find the area of the shaded region bounded by the curve, the lines $y = 1$ and $y = 8$ and the y-axis.

3



The diagram shows a solid triangular prism. The cross-section of the prism is an equilateral triangle of side x cm and the length of the prism is l cm.

Given that the volume of the prism is 250 cm^3,

a find an expression for l in terms of x,

b show that the surface area of the prism, A cm^2, is given by

$$A = \frac{\sqrt{3}}{2}\left(x^2 + \frac{2000}{x}\right).$$

Given that x can vary,

c find the value of x for which A is a minimum,

d find the minimum value of A in the form $k\sqrt{3}$,

e justify that the value you have found is a minimum.