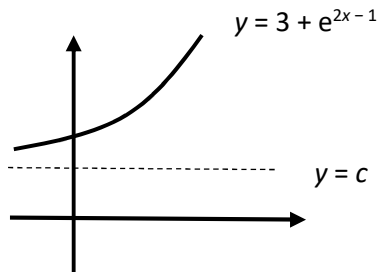


## Pure 23 - Linearising

Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

### Section 1 – Review of previous topics. Please complete all questions.

1.



The diagram shows the curve with the equation  $y = 3 + e^{2x-1}$  and the asymptote of the curve which has the equation  $y = c$ .

- State the value of the constant  $c$ .
- Find the exact coordinates of the point where the curve crosses the  $y$ -axis.
- Find the  $x$ -coordinate of the point on the curve where  $y = 7$ , giving your answer in the form  $a + \ln b$ , where  $a$  is rational and  $b$  is an integer.

2. Solve each equation, giving your answer in exact terms.

- a)  $\ln x = 15$       b)  $\ln(\frac{1}{2}x + 3) = 2.5$       c)  $e^x = 0.7$       d)  $e^{4t+1} = 12$       e)  $2e^{2x} + 12 = 11e^x$   
 f)  $\ln(3x^2 - 10x + 8) - \ln(x^2 - 5x + 6) = \ln 2x$

### Section 2 – Consolidation of this week's topic. Please complete all questions.

1. The population of a country,  $P$  millions, in  $t$  years' time is modelled by the formula  $P = Ac^t$  where  $A$  and  $c$  are constants.

- a) Show that a graph of  $\log P$  against  $t$  is a straight line. (5)

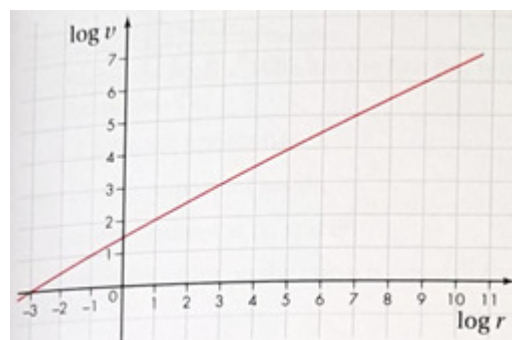
The gradient of the straight line is 0.0128 and the intercept on the  $\log P$  axis is (0, 1.97).

- Find the values of  $A$  and  $c$ . (5)
- Find the annual rate of growth of the population. (2)
- Explain why the model might not be accurate for a large value of  $t$ . (1)

2. The formula connecting two variables,  $v$  and  $t$ , is  $v = ar^n$  where  $a$  and  $n$  are constants.

Here is a graph of  $\log v$  against  $\log r$ :

- Find the gradient of the graph. (2)
- Explain why  $\log a = 1.5$ . (4)
- Find the values of  $a$  and  $n$ . (3)
- Use the formula to find the value of  $v$  when  $r = 100$ . (2)



3. Below is data for the moons of Saturn.

It is believed that  $R = kT^n$ , where  $R$  = radius of moon and  $T$  = time to orbit Saturn.

Moon	Tethys	Dione	Rhea	Titan	Iapetus
Radius, $R$ ( $\times 10^5$ km)	2.9	3.8	5.3	12.2	35.6
Period, $T$ (days)	1.9	2.7	5.4	15.9	79.3

a) Show that  $\log R = \log k + n \log T$  (3)

b) Plot a graph of  $\log T$  on the horizontal axis and  $\log R$  on the vertical axis. (4)

c) Measure the gradient of the line of best fit and use this to calculate  $n$ . Also calculate  $k$  from your graph. (5)

d) A new moon was discovered to have a radius of  $1.4 \times 10^5$  km, estimate how many days it takes to orbit Saturn. (4)

**TOTAL 40 MARKS**

**Section 3 – Extension questions.** If you are aiming for a top grade, you should attempt these questions.

### MOORE'S LAW

Moore's law relates the number of transistors ( $T$ ) that can be fitted on a computer chip to the number of years from 1971 ( $t$ ). It is of the form  $T = ae^{kt}$

Using the data in the table below, plot a graph of  $\ln(T)$  against  $t$  and use your graph to find the values of  $a$ , and  $k$ . Use your formula to predict the number of transistors on a computer chip in 2010. Check and comment on your answer by googling the number of transistors on a computer chip in 2010.

(NOTE: if you use a spreadsheet such as Excel it can do the calculations for you and plot the graph)

Date	$t$	Transistors ( $T$ )
1971	0	2,300
1972	1	2,500
1974	3	4,500
1978	7	29,000
1982	11	134,000
1985	14	275,000
1989	18	1,200,000
1993	22	3,100,000
1997	26	7,500,000
1999	28	9,500,000
2000	29	42,000,000
2002	31	220,000,000
2004	33	592,000,000