

Pure 26 - Sigma notation and recurrence relations

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Consolidation of this week's topic. Please complete all questions.

1. List the first 4 terms of the sequence defined by $u_{n+1} = 2u_n + 3$, $u_1 = 2$. (2)

2. For each sequence given, list the first four terms and hence state whether the sequence is convergent or divergent.
 - a) $u_{n+1} = 4u_n - 1$, $u_1 = 2$ (2)
 - b) $u_n = 3 + 0.2^n$ (2)

3. Write out fully the series given:
 - a) $\sum_{r=1}^4 (2r + 3)$ b) $\sum_{r=0}^3 2 \times 3^r$ c) $\sum_{r=1}^4 (-1)^r r^2$ (3)

4. The n^{th} term of an arithmetic sequence is u_n where $u_n = 45 - 6n$.
 - a) Find the value of u_1 and the value of u_2 . (1)
 - b) Write down the common difference of the arithmetic sequence. (1)
 - c) Given that $\sum_{r=1}^n u_r = 0$, find the value of n . (3)

5. The n^{th} term of a sequence is given by $u_n = 17 - 4n$
 - a) Find the first 3 terms (2)
 - b) Find the sum of the first 12 terms of the sequence (3)
 - c) Write the sum of the first 20 terms in sigma notation (3)
 - d) Show that the sum of the first n terms = $n(15 - 2n)$ (3)

6. Evaluate (find the value of) by showing your working:
 - a) $\sum_7^{10} (2r - 1)$ (2)
 - b) $\sum_{r=1}^{20} (1 + 3r)$ (2)

7. Write down the first four terms of each sequence.
 - a) $u_n = u_{n-1} + 4, n > 1, u_1 = 3$ b) $u_n = 3u_{n-1} + 1, n > 1, u_1 = 2$

c $u_{n+1} = 2u_n + 5, n > 0, u_1 = -2$

d $u_n = 7 - u_{n-1}, n > 1, u_1 = 5$

e $u_n = 2(5 - 2u_{n-1}), n > 1, u_1 = -1$

f $u_n = \frac{1}{10}(u_{n-1} + 20), n > 1, u_1 = 10$

(6)

8. In each case, write down a recurrence relation that would produce the given sequence.

a 5, 9, 13, 17, 21, ...

b 1, 3, 9, 27, 81, ...

c 62, 44, 26, 8, -10, ...

d 120, 60, 30, 15, 7.5, ...

e 4, 9, 19, 39, 79, ...

f 1, 3, 11, 43, 171, ...

(6)

9. A sequence is given by the recurrence relation $u_n = \frac{1}{2}(k + 3u_{n-1}), n > 1, u_1 = 2$

a Find an expression for u_3 in terms of the constant k .

Given that $u_3 = 7$,

(2)

b Find the value of k and the value of u_4 .

(2)

10. Find the sum of

a all even numbers between 2 and 160 inclusive,

(1)

b all positive integers less than 200 that are divisible by 3,

(1)

c all integers divisible by 6 between 30 and 300 inclusive.

(2)

11. The fifth, sixth and seventh terms of an arithmetic series are $(5 - t)$, $2t$ and $(6t - 3)$ respectively.

a Find the value of the constant t .

(2)

b Find the sum of the first 18 terms of the series.

(2)

12. The first three terms of a geometric series are $(k - 8)$, $(k + 4)$ and $(3k + 2)$ respectively, where k is a positive constant.

a Find the value of k .

(2)

b Find the sixth term of the series.

(2)

c Show that the sum of the first ten terms of the series is 50,857.3 to 1 decimal place

(1)

13. When a ball is dropped onto a horizontal floor it bounces such that it reaches a maximum height of 60% of the height from which it was dropped.

a Find the maximum height the ball reaches after its fourth bounce when it is initially dropped from 3 metres above the floor.

(2)

b Show that when the ball is dropped from a height of h metres above the floor it travels a total distance of $4h$ metres before coming to rest.

(3)

Total 63 marks