Pure 27 – Partial Fractions

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.
Please <u>complete</u> all questions.

1. The fifth term of an arithmetic series is 23 and the sum of the first 10 terms of the series is 240.
   a) Find the first term and common difference of the series
   b) Find the sum of the first 60 terms of the series

2. Given that $\sum_{r=1}^{n}(4r - 6) = 720$, find the value of n

3. The second and fifth terms of a geometric series are 0.5 and 32 respectively.
   a) Find the first term and the common ratio of the series
   b) Find the number of terms of the series that are smaller than 10000

4. The common ratio of a geometric series is 0.55 and the sum to infinity of the series is 40.
   a) Find the first term of the series
   b) Find the smallest value of n for which the nth term of the series is less than 0.001

5.
   a) Describe the transformation required to map the graph of $y = f(x)$ onto $y = f(-x)$
   b) Describe the transformation required to map the graph of $y = f(x)$ onto $y = -f(x)$
   c) Given that $f(x) = 3x + 2$, state the equations of $f(-x)$ and $-f(x)$

6. $f(x) = 3x + 2$
   a) On the same axes sketch the graphs of $y = 5, y = f(x)$ and $y = f(-x)$
   b) Find the three coordinates where two of these lines intersect
   c) What do you notice?

7. What would the denominator be when expressing the following as simplified single fractions:
   a) $\dfrac{2}{x+3} - \dfrac{4}{x+2}$

   b) $\dfrac{6x+1}{x^2} + \dfrac{3}{x}$

   c) $\dfrac{5}{x} - \dfrac{2}{x+1} + \dfrac{3}{x+2}$

8.
   a) Show that $x + 3$ is a factor of $2x^3 + x^2 - 13x + 6$
   b) Hence express $2x^3 + x^2 - 13x + 6$ as the product of three linear factors
   c) Hence express $\dfrac{2x^3+x^2-13x+6}{x^2-2x}$ in the form $Ax + B + Cx^{-1}$, where A, B and C are constants to be determined.

Section 2 – Consolidation of this week's topic.
Please complete all questions.

1. Express in partial fractions

   a) $\dfrac{8}{(x-1)(x+3)}$

   c) $\dfrac{1-3x}{(3x+4)(2x+1)}$

   b) $\dfrac{5x+7}{x^2+x}$

   d) $\dfrac{2(x+5)}{8x^2+10x-3}$

   (3 marks each)

2. Find the values of the constants A, B and C in the identities below:

   a) $\dfrac{8x+14}{(x-2)(x+1)(x+3)} \equiv \dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{x+3}$

   b) $\dfrac{2x^2-6x+20}{(x+1)(x-2)(x-6)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x-2} + \dfrac{C}{x-6}$

   (3 marks each)

3. Express in partial fractions

   a) $\dfrac{9}{(x-2)(x+1)^2}$

   b) $\dfrac{5x^2+3x-20}{x^3+4x^2}$

   (4 marks each)

4. Find the values of the constants A, B and C in the identities below:

   a) $\dfrac{x^2}{(x-2)(x-6)} \equiv A + \dfrac{B}{x-2} + \dfrac{C}{x-6}$

   b) $\dfrac{x^2+2x+9}{x^2+4x-5} \equiv A + \dfrac{B}{x-1} + \dfrac{C}{x+5}$

   (4 marks each)

5.

   a) $\dfrac{5-12x}{(1+6x)(4+3x)} \equiv \dfrac{A}{(1+6x)} + \dfrac{B}{(4+3x)}$. Find A and B

   b) Find the binomial expansions of $(1+6x)^{-1}$ and $(4+3x)^{-1}$, up to and including the term in x^2.

   c) Hence find the binomial expansion of $\dfrac{5-12x}{(1+6x)(4+3x)}$, up to and including the term in x^2

   d) For what values of x is this expansion valid?

   (10 marks)
   Total 44 Marks