

## Pure 28 – Proof

Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

### Section 1 – Review of previous topics.

Please complete all questions.

- It is suggested that the sequence  $a_k = 2^k + 1$ ,  $k \geq 1$  produces only prime numbers.
  - Verify that  $a_1$ ,  $a_2$  and  $a_4$  produce prime numbers.
  - Prove by counter example that the sequence does not always produce a prime number.
- Mo is training for a 10km race. On the first day he runs 2km. He schedules his training to increase by 0.5km each day so that he runs 2.5km on the second day, 3km on the third and so on. This continues until he reaches the maximum distance of 10km. He then runs 10km each day until (and including) the twentieth day. What is the total distance that Mo will run in training?
- When a baby is born, £3000 is invested into an account with a fixed interest rate of 4% per year.
  - What will the account be worth at the start of the seventh year?
  - After how many full years will the account have doubled in value?
- $a, ar, ar^2, \dots$  is a geometric sequence. Given that the sum to infinity is  $2a$ , find  $r$ .
- $f(x) = 2x^3 - x^2 - 13x - 6$  Use the factor theorem and division to factorise  $f(x)$  completely.
- A circle has the equation  $x^2 + y^2 + 2x + 4y = 5$ . The point  $A(0, -5)$  lies on the circle. Find the tangent to the circle at  $A$  in the form  $ax + by = c$
- Sketch the graphs of the equations below, labelling the coordinates of any stationary points and axes intercepts.
  - $y = x^4 - x^2$
  - $y = -x^3 + 9x$

Section 2 – Consolidation of this week's topic.  
Please complete all questions.

Prove the following taking care to use the specified method;

1. There is no largest multiple of three. (by contradiction) (3)
2. The product of two odd numbers is odd. (by contradiction) (3)
3. If  $n$  is odd then  $n^3 + 1$  is even. (by deduction) (3)
4. There is no greatest positive rational number. (by contradiction) (3)
5. If  $n^2$  is even, then  $n$  must be even. (by contradiction) (3)
6.  $\sqrt{2}$  is irrational. (by contradiction) (5)
7.  $1 + \sqrt{2}$  is irrational, given that  $\sqrt{2}$  is known to be irrational. (by contradiction) (3)
8. There exist no integers  $a$  and  $b$  for which  $25a + 15b = 1$  (by deduction) (3)
9. There are infinitely many prime numbers. (by contradiction) (4)
10. Prove that the sum of the first  $n$  terms of an arithmetic sequence is  $\frac{n}{2}[2a + (n - 1)d]$  where  $a$  is the first term and  $d$  is the common difference between terms. (4)
11. Prove that the sum of the first  $n$  terms of a geometric sequence is  $\frac{a(1-r^n)}{1-r}$  where  $a$  is the first term and  $r$  is the common ratio between terms. (4)

**Total: 38 Marks**