

Pure 30 – Functions: domain, range, composites and inverse

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Express $\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$ as a single fraction in its simplest form.

2. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a , b , c , d and e .

3. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

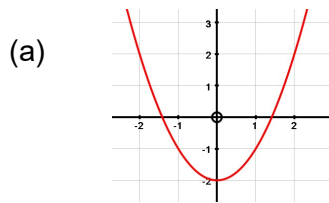
(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

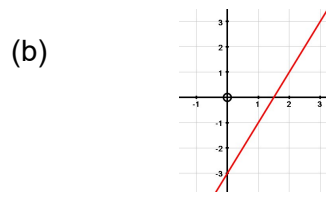
Show, on each diagram, the point where the graph meets or crosses the x -axis.
In each case, state the equation of the asymptote.

Section 2 – Consolidation of this week’s topic. Please complete all 10 questions.

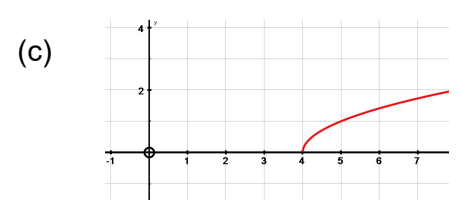
1. State the largest possible domain and range for each function shown:



$$f(x) = x^2 - 2$$



$$f(x) = 2x - 3$$



$$f(x) = \sqrt{x - 4} \quad (6)$$

2. The function f is defined as $f(x) = 4x - 5$, $x \in \mathbb{R}$. Find

- a. $f(3)$ b. $f(-2)$ c. $f(\frac{1}{4})$ d. the inverse function $f^{-1}(x)$ (5)

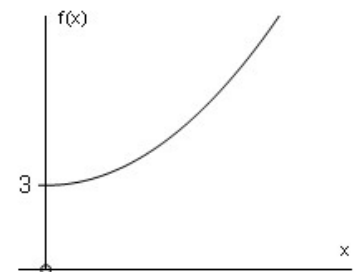
3. Given $f(x) = x^3$, $x \in \mathbb{R}$ and $g(x) = 4x - 1$, $x \in \mathbb{R}$.

Find the following:

- a. $fg(x)$ b. $gf(x)$ c. $gg(x)$ (4)

4. The graph of $f(x) = x^2 + 3$, $x \geq 0$, $x \in \mathbb{R}$ is shown.

- a. State the range of $f(x)$.
 b. Find the inverse function f^{-1} , stating its domain.
 c. Sketch the graph of $f(x)$ and $f^{-1}(x)$ on the same diagram.
 d. State the transformation which maps $y = f(x)$ onto $y = f^{-1}(x)$. (6)



5. Find f^{-1} , the inverse of function f , where $f: x \rightarrow \frac{x}{x+5} - 3$, $x \neq -5$, stating its domain. (4)

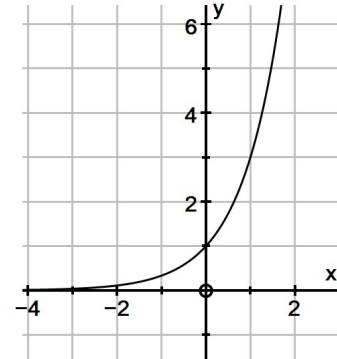
6. Two functions f and g are defined by $f(x) = 5x + 6$ and $g(x) = \frac{2}{x}$, ($x \neq 0$)

- a. find rules for $fg(x)$ and $gg(x)$
- b. find a rule for $(fg)^{-1}(x)$

(5)

7. The graph of $f(x) = 3^x$, $x \in \mathbb{R}$ is shown.

- i. State the range of $f(x)$
- ii. Find the **exact** value of $f(-2)$.
- iii. State whether $f(x)$ is a one-to-one function or a many-to-one function
- iv. Find an expression for $f^{-1}(x)$, stating its domain.



(5)

8. $f(x) = \sqrt{x+4}$, $x \geq -4$, $x \in \mathbb{R}$

- a. Sketch the graph of $y = f(x)$
- b. State the range of $f(x)$
- c. Find the inverse function $f^{-1}(x)$ and state its domain.

(5)

9. The functions f and g are defined by

$$f : x \rightarrow 7x - 1, \quad x \in \mathbb{R},$$

$$g : x \rightarrow \frac{4}{x-2}, \quad x \neq 2, x \in \mathbb{R},$$

(a) Solve the equation $fg(x) = x$.

(4)

(b) Hence, or otherwise, find the largest value of a such that $g(a) = f^{-1}(a)$.

(1)

(Total 5 marks)

10.

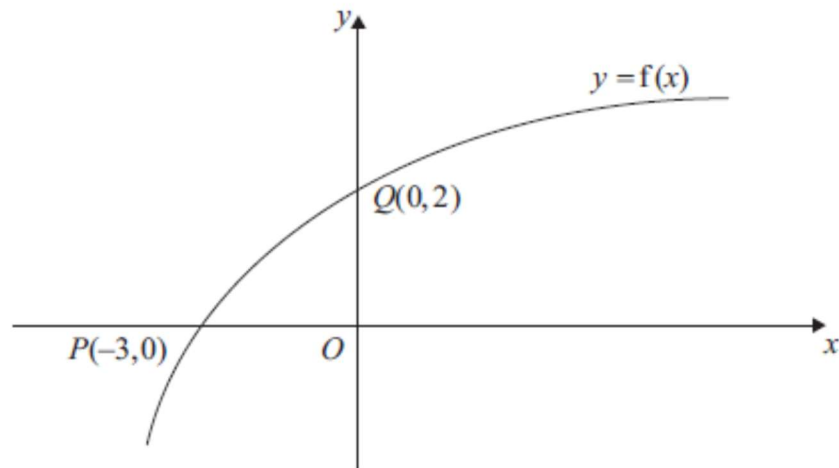


Figure 3

Figure 3 shows part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(a) Find the value of $ff(-3)$. (2)

On separate diagrams, sketch the curve with equation

(b) $y = f^{-1}(x)$, (2)

(c) $y = f(|x|) - 2$, (2)

(d) $y = 2f\left(\frac{1}{2}x\right)$. (3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

(Total 9 marks)

Total: 54 marks