Pure 33 – Trig Equations and Proofs

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 3x)^5,$$

giving each term in its simplest form.

2. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.
Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,
(b) find the value of b.

3. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1 + \frac{x}{4}\right)^8,$$

giving each term in its simplest form.
(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

4. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4,$$

giving each term in its simplest form.

$$f(x) = (1 + kx)(2 - 9x)^4, \quad \text{where } k \text{ is a constant.}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2,$$

where A and B are constants.
(b) Write down the value of A.
(c) Find the value of k.
(d) Hence find the value of B.

Section 2 – Consolidation of this week's topic. Please complete all questions. (Total 58 marks)

1) Solve each equation for θ in the interval $0 \leq \theta \leq 2\pi$

a) $3sec^2\theta = 4tan^2\theta$ (4 marks)

b) $cot^2\theta - 3cosec\theta + 3 = 0$ (4 marks)

c) $sec^2\theta + 2tan\theta = 0$ (3 marks)

2) Solve each equation for x in the interval $-180° \leq x \leq 180°$

a) $tan^2x - 2secx - 2 = 0$ (5 marks)

b) $cosec^2x + 5cosecx + 2cot^2x = 0$ (5 marks)

c) $tan^2x + 4secx - 2 = 0$ (4 marks)

3) Solve each equation in the interval $0 \leq x \leq 360°$

a) $cot^2 2x + cosec2x - 1 = 0$ (6 marks)

b) $3cosec^2x - 4sin^2x = 1$ (5 marks)

4) Prove each of the following identities

a) $cosec^2x - sec^2x \equiv cot^2x - tan^2x$ (2 marks)

b) $(cosx - 2secx)^2 \equiv cos^2x + 4tan^2x$ (3 marks)

c) $(tanx + cotx)^2 \equiv sec^2x + cosec^2x$ (3 marks)

d) $sec^2x + cosec^2x \equiv sec^2xcosec^2x$ (4 marks)

5) a) Given that $secx + tanx = -3$, use the identity $1 + tan^2x \equiv sec^2x$ to find the value of $secx - tanx$. (3 marks)

b) Deduce the values of:
(i) $secx$ (ii) $tanx$ (3 marks)

c) Hence solve, in the interval $-180° \leq x \leq 180°, secx + tanx = -3$ (3 marks)