Godalming College

# Pure 13 – Parametrics

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

## Section 1 – Review of previous topics. Please complete all questions.

**1.** Given that the binomial expansion of $(1 + kx)^{-4}$, $|kx| < 1$, is

$$1 - 6x + Ax^2 + \ldots$$

(*a*) find the value of the constant $k$,

(*b*) find the value of the constant $A$, giving your answer in its simplest form.

**2.** (*a*) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R$ and $\alpha$ are constants, $R > 0$ and $0 < \alpha < 90°$ Give the exact value of $R$ and give the value of $\alpha$ to 2 decimal places.

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(*c*) Use your solutions to parts (*a*) and (*b*) to deduce the smallest positive value of $\theta$ for which

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

**3.** (*a*) Express $4\,\text{cosec}^2 2\theta - \text{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

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(b) Hence show that

$$4\,\text{cosec}^2 2\theta - \text{cosec}^2\theta = \sec^2\theta .$$

(c) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4\,\text{cosec}^2 2\theta - \text{cosec}^2\theta = 4$$

giving your answers in terms of $\pi$.

## Section 2

### Exercise

1. Find the Cartesian equations of the curves given by
   (i) $x = 1 - t,\ y = t^2 - 4$
   (ii) $x = 2t^2,\ y = \dfrac{1}{t}$
   (iii) $x = 2\cos\theta + \sin\theta,\ y = \cos\theta - 2\sin\theta$ [9]

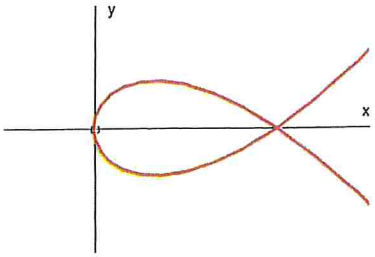
2. A curve has parametric equations $x = t^2, y = t^3$.
   (i) Calculate values for $x$ and $y$ for values of $t$ between –3 and +3.
   (ii) Sketch the curve.
   (iii) Find the Cartesian equation of the curve. [10]

3. A curve has parametric equations $x = 3\cos\theta,\ y = 2\sin\theta$.
   (i) Calculate values for $x$ and $y$ for values of $\theta$ from 0 to $\pi$, at intervals of $\dfrac{\pi}{12}$.
   (ii) Using what you know about angles greater than $\pi$ calculate values for $x$ and $y$ for $\theta$ from $\pi$ to $2\pi$.
   (iii) Sketch the curve.
   (iv) Find the Cartesian equation of the curve. [16]

4. A curve has parametric equations $x = t + t^2,\ y = t - t^2$.
   (i) Find the values of $t$ for which the curve meets the $x$-axis. [3]

5. Given the parametric equations $x = t - \dfrac{1}{t},\ y = 2\left(t + \dfrac{1}{t}\right)$,
   (i) For what value of $t$ is the curve undefined?
   (ii) Find the coordinates of any points where the curve meets the coordinate axes.
   (iii) Find the Cartesian equation. [11]

6. The curve shown below has parametric equations $x = 4t^2$, $y = 2t(1-t^2)$.




(i) By eliminating $t$, find the Cartesian equation of the curve. (3)

7. A ball is struck at ground level and projected with a speed of 16 $ms^{-1}$ at an angle $\theta$ to the horizontal. The parametric equations of the path of the ball are given by $x = 16t\cos\theta$, $y = 16t\sin\theta - 5t^2$.

(i) By eliminating $t$ show that the Cartesian equation of the path can be written as a quadratic in $\tan\theta$.

Given that $\theta = 30°$,

(ii) How far does the ball travel horizontally before bouncing? (9)

TOTAL: 61