

## Pure 39 – Differentiation: Chain Rule, Product & Quotient Rules

Please **complete** this homework by \_\_\_\_\_. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

### Section 1 – Review of previous topics.

Please complete all questions.

1. Find the equation of the normal to the curve  $y = x^2 - \frac{8}{\sqrt{x}}$  at the point (4,12).
2. Find the values of  $x$  for which  $f(x) = 3x^2 + 8x + 2$  is an increasing function.
3. Given that  $y = 3x^5 + \frac{4}{x^2}$ , find  $\frac{d^2y}{dx^2}$ .
4.  $f(x) = px^3 - 3px^2 + x^2 - 4$ . When  $x = 2$ ,  $f''(x) = -1$ . Find the value of  $p$ .
5. For the function  $f(x) = 2x^3 - 15x^2 + 24x + 6$ , find:
  - a) The coordinates of the stationary points
  - b) The nature of these stationary points (ie whether they are minimum or maximum points)
6. Given that  $x \neq 0$  find the set of values for which  $5 > \frac{4}{x}$ . Give your answer using set notation.
7. The equation  $kx^2 - 2kx + 3 = 0$  where  $k$  is a constant has no real roots. Prove that  $k$  satisfies the inequality  $0 \leq k < 3$ .
8. A straight line passes through the points  $(a, 4)$  and  $(3a, 3)$ . Its equation is  $x + 6y + c = 0$ . Find the values of  $a$  and  $c$ .
9. A line is perpendicular to the line  $3x + 8y - 11 = 0$  and passes through the point  $(0, -8)$ . What is the equation of the line in the form  $ay + bx + c = 0$ ?
10. A point  $P$  lies on the line with equation  $y = 4 - 3x$ . The point  $P$  is a distance  $\sqrt{34}$  from the origin. Find the two possible positions of point  $P$ .

## Section 2 – Consolidation of this week’s topic.

Please complete all questions.

**1)** Differentiate with respect to  $x$  and simplify where possible:

a)  $y = e^{x^2}$       b)  $f(x) = \sin^2 x$       c)  $y = e^{2 \sin x}$   
d)  $f(x) = \sin(\sqrt{x}) + \sqrt{\sin x}$       e)  $y = 2 \ln(3x + 5)$       **[6]**

**2)** Differentiate with respect to  $x$  and simplify where possible:

a)  $y = 5x(3x^2 + 1)^3$       b)  $f(x) = \sin 4x \cos 5x$       c)  $y = \ln 2x \sin 3x$       **[7]**

**3)** Differentiate with respect to  $x$  and simplify where possible:

a)  $y = \frac{x}{x^2 - 1}$       b)  $f(x) = \frac{e^x}{\cos x}$       c)  $y = \frac{\sqrt{x}}{\sqrt{x} + 1}$       **[5]**

**4)** Differentiate with respect to  $x$  and simplify where possible:

a)  $y = x^2 e^{-3x}$       b)  $f(x) = \sqrt{x} \ln 3x$       c)  $y = x \ln \left( \frac{x-1}{x+1} \right)$   
d)  $f(x) = \ln(\cos x)$       **[11]**

**5)** A curve has the equation  $y = xe^{x^2}$ .      **[8]**

- a) Find the equation of the tangent to the curve at  $x = 1$ .
- b) Find the coordinates of the points where the tangent cuts the  $x$  and  $y$  axes.
- c) Find the area of the triangle bounded by this tangent and the  $x$  and  $y$  axes.

**6)** The curve  $y = (x + 3)(x - 1)^3$  crosses the  $x$  axis at 2 points P and Q and has a turning point at R. Given that the  $x$  coordinate of P is less than the  $x$  coordinate of Q, write down the coordinates of P and Q and find the coordinates of R, stating whether it is a minimum or maximum.      **[11]**

**7)**  $f(x) = x\sqrt{2x + 12}$ ,  $x \geq -6$ .      **[13]**

- a) Find  $f'(x)$
- b) Show that  $f''(x) = \frac{3(x+8)}{(2x+12)^{\frac{3}{2}}}$ .
- c) Find the turning point of the curve and determine its nature.

**Total: 61 Marks**