Pure 40 – Differentiation: Trig Functions & Parametric Functions

Please **complete** this homework by ___________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.
Please complete all questions.

1. A possible point of inflection is where the second derivative of a function is equal to zero. Identify the stationary point of $y = x^3 - 3x^2 + 3x$ and show that this point is a possible point of inflection.

2. The curve with equation $y = \frac{1}{x} + 27x^3$ has stationary points at $x = \pm a$. Find the value of a and determine the nature of the stationary points.

3. The coordinates of points A and B are (-4, 6) and (2, 8) respectively. Find the equation of the perpendicular bisector of AB.

4. The perpendicular bisector of the line segment joining (5, 8) and (7, -4) crosses the x axis at the point Q. Find the coordinates of Q.

5. A triangle has sides of length 5, x and x + 2. The side of length 5 is opposite to an angle of 60°. Find x to 3 significant figures.

6. Without using a calculator, if $\sin\theta = \frac{3}{5}$, what are $\cos\theta$ and $\tan\theta$?

7. Simplify the following trig expressions:

a) $\sin^2 3\theta + \cos^2 3\theta$ b) $\frac{\sin 2\theta}{\sqrt{1-\sin^2 2\theta}}$

8. Given that $0 < \theta \leq 360°$, solve $7\sin\theta = 5$.

9. Given that $0 \leq \theta \leq 180°$, solve $\sin(3\theta - 45°) = \frac{1}{2}$.

10. Given that $0 \leq \theta \leq 180°$, solve $2\sin^2\theta = 3(1 - \cos\theta)$.

Section 2 – Consolidation of this week's topic.
Please complete all questions.

1) Differentiate with respect to x and simplify where possible:

a) $y = 2\sec x$ b) $f(x) = \text{cosec}\,4x$ c) $y = 5\cot(\frac{\pi x}{3})$

d) $f(x) = 3\sec(x-3)$ e) $y = \cot(2x-3)$ **[10]**

2) Show that the curve with equation $y = e^x \cot x$ has no turning points. **[5]**

3) A curve has the equation $x = \tan^2 y$.

a) Show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x}(x+1)}$

b) The equation of the normal to the curve at the point where $y = \frac{\pi}{4}$ has a negative gradient. Find the equation of this normal. **[7]**

4) Differentiate with respect to x and simplify where possible:

a) $y = \sec^2 2x$ b) $f(x) = \cot^3 x$ c) $y = \text{cosec}^2(2x+1)$

d) $f(x) = \ln(\tan 4x)$ e) $y = e^{\sin 3x}$ **[10]**

5) A curve has the equation $y = \text{cosec}\left(x - \frac{\pi}{6}\right)$ and crosses the y axis at the point P. The point Q on the curve has x coordinate $\frac{\pi}{3}$.

a) Find an equation for the normal to the curve at P.

b) Find an equation for the tangent to the curve at Q.

The normal to the curve at P and the tangent to the curve at Q meet at R.

c) Show that the x coordinate of R is $\frac{8\sqrt{3}+4\pi}{13}$. **[11]**

6) Find $\frac{dy}{dx}$ in terms of the parameter t:

a) $x = t^3$ $y = t$ b) $x = 3t - 1$ $y = 2 - \frac{1}{t}$

c) $x = \cos 2t$ $y = \sin t$ d) $x = e^{t+1}$ $y = e^{2t-1}$ **[7]**

7) A curve is given by the parametric equations $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$ $(t \neq 0)$.

a) Find an equation for the tangent to the curve at the point P where $t = 3$.

b) Show that the Cartesian equation of the curve is $x^2 - y^2 = k$ where k is a constant to be found.

c) Show that the tangent to the curve at P does not meet the curve again. **[10]**

Total: 60 Marks