

Pure 41 – Differentiation: Implicit, Second Derivative & Rates of Change

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1. The surface area of an expanding sphere of radius r is given by $4\pi r^2$. Find the rate of change of the area with respect to the radius when $r = 6$ cm.
2. Differentiate $2x^3 + \sqrt{x} + \frac{x^2+2x}{x^2}$.
3. The point $(1, -3)$ lies on the circle $(x - 3)^2 + (y + 4)^2 = r^2$. Find the value of r .
4. The line with equation $y = 4x - 1$ does not intersect the circle with equation $x^2 + 2x + y^2 = k$. Find the range of possible values of k .
5. The points $(-2, 8)$, $(7, 7)$, $(-3, -1)$ lie on a circle. Find the equation of the circle.
6. Use the factor theorem to show whether $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.
7. Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$, find the value of a .
8. Use algebraic division to find the cubic polynomial that arises from $(3x^4 + 8x^3 - 11x^2 + 2x + 8) \div (3x + 2)$.
9. It is asserted that $a + \frac{1}{a} \geq 2$. Prove that the inequality is true only if $a > 0$.
10. Prove that, for any distinct positive numbers p and q , $p + q > \sqrt{4pq}$.

Section 2 – Consolidation of this week’s topic.

Please complete all questions.

1) Differentiate these implicitly:

a) $x^2 + y^2 = 3$ b) $2x - y + y^2 = 5$ c) $\sin x + \cos y = 1$
d) $2e^x - 3e^{2y} = y$ e) $\ln 2x + 3 \ln y^2 = x$ [10]

2) Find the equation of the tangent to $4 \sin y - \sec x = 0$ at $(\frac{\pi}{3}, \frac{\pi}{6})$. [5]

3) Differentiate these implicitly:

a) $x^3y = 5$ b) $xe^{2y} - \ln xy = 2$ c) $x \sin y + y^2 \operatorname{cosec} x = 0$
d) $xy - \sin x = e^y$ e) $\ln(x + 2) = \ln(2y + 1)$ [16]

4) A curve has the equation $3^x + y^2 = (x + 3)y$. Find the equation of the normal to the curve at the point (1,1). [5]

5) Find the points of inflection of these functions and determine the intervals over which they are concave or convex.

a) $y = x^4 - 54x^2$ b) $y = xe^{-x}$ [11]

6) Given that $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$. [3]7) The volume of water in a vase is given by $V = 10\pi(e^{0.2h} - 1)$ where h = depth of water in the vase. The depth of water is increasing at a rate of 0.6 cm s^{-1} . What is the rate of increase of the volume when the depth is 5cm? [4]8) An inverted cone is being filled with sand at a rate of $10 \text{ cm}^3 \text{ s}^{-1}$. After 5 seconds the depth of the sand is 20cm. What is the rate of increase of the depth of sand when the depth of sand is 10cm? (Hint: $V = \frac{1}{3}\pi r^2 h$ where both r and h are variables but we can put $\tan \theta = \frac{r}{h}$ and, since θ is not variable, we can substitute for r and reformulate the expression for V with only h as a variable). [6]9) A biological culture is growing exponentially such that the number of bacteria present N at time t minutes is given by $N = 500 \times 1.05^{0.4t}$. Find the rate at which the bacteria are growing when $N = 2000$. [3]

Total: 63 Marks