

Pure 44 – Integration: Substitution

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.

Please complete all questions.

- 1) a) Solve the inequalities. You must show your working.
 - i) $3x - 5 < 11 - x$
 - ii) $x^2 - 6x + 5 \leq 0$
 b) Show on a graph the set of values of x that satisfy both $3x - 5 < 11 - x$ and $x^2 - 6x + 5 \leq 0$

- 2) The graph of $y = ab^x$ passes through the points $(0, 5)$ and $(2, 1.25)$
 - a) Find exact values for a and b
 - b) Sketch the curve

- 3) Use proof by contradiction to prove that, if n is an integer, and n^n is odd, then n is odd.

- 4) The second term of a geometric series is 120 and the fifth term is 15. Work out
 - a) The common ratio of the series
 - b) The first term of the series
 - c) The sum to infinity of the series

- 5) Show that the curve with Cartesian equation $\frac{x^2}{25} - \frac{y^2}{9} = 1$ has parametric equations $x = 5 \sec \theta$, $y = 3 \tan \theta$

- 6) A cuboid has length twice its width as shown
The volume of the cuboid is 192 cm^3
 - a) Show that the surface area of the cuboid, S , is given by $S = 4x^2 + \frac{k}{x}$, where k is a constant to be found
 - b) Find the minimum value of S , showing your working
 - c) Use calculus to justify that this is a minimum



Section 2 – Consolidation of this week’s topic.

Please complete all questions.

1) Using the given substitution, find:

a) $\int x(2x - 1)^4 dx$	$u = 2x - 1$	b) $\int x\sqrt{1-x} dx$	$u = 1 - x$
c) $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$	$x = \sin u$	d) $\int \frac{1}{\sqrt{x-1}} dx$	$x = u^2$
e) $\int (x + 1)(2x + 3)^3 dx$	$u = 2x + 3$	f) $\int \frac{x^2}{\sqrt{x-2}} dx$	$u^2 = x - 2$ [18]

2) Using the given substitution, evaluate:

a) $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$	$x = \sin u$	b) $\int_0^2 x(2-x)^3 dx$	$x = 2 - u$
c) $\int_0^1 \sqrt{4-x^2} dx$	$x = 2 \sin u$	d) $\int_0^3 \frac{x^2}{x^2+9} dx$	$x = 3 \tan u$ [16]

3) Use the substitution $u^2 = e^x - 2$ to show that $\int_{\ln 3}^{\ln 4} \frac{e^{4x}}{e^{x-2}} dx = \frac{a}{b} + c \ln d$, where a, b, c and d are integers to be found. **[8]**

4) Using a suitable trigonometric substitution for x, find $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x^2 \sqrt{1-x^2} dx$ **[8]**

Total: 50 Marks