Pure 47 – Integration: Partial Fractions and Trapezium Rule

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics. Please complete all questions.

1) The equation of a circle is $x^2 + y^2 - 10x + 2y - 23 = 0$

a) Showing your working clearly, work out

i) Its centre ii) its radius

b) The line $y = x + 2$ meets the circle at the points P and Q. Work out, in exact form, the coordinates of P and Q.

2) Given vectors $\mathbf{p} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

a) Evaluate $3\mathbf{p} + 5\mathbf{q}$

b) Write down the unit vector, $\hat{\mathbf{q}}$ in the direction of $\mathbf{q}$

3) Write the Cartesian equation of the curve that is given parametrically by

$$x = \frac{1}{2t+1}, y = \frac{2}{3-t}, t > 3$$

4) a) Show that $\sec^4 x - \tan^4 x \equiv \sec^2 x + \tan^2 x$

b) Find the values in the range $-\pi \leq x \leq \pi$ that satisfy $\sec^4 x - \tan^4 x \equiv 5 + \tan^2 x$. Show your working.

5) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ given that $5xy - y^3 = 7$

6) Use implicit differentiation to prove that the derivative of a^x is $a^x \ln a$

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Section 2 – Consolidation of this week's topic.

Please complete all questions.

1) a) Express $\frac{3x+5}{(x+1)(x+3)}$ in partial fractions

b) Hence, find $\int \frac{3x+5}{(x+1)(x+3)}\,dx$ **[8]**

2) Show that $\int \frac{3}{(t-2)(t+1)}\,dt = \ln\left|\frac{t-2}{t+1}\right| + c$ **[7]**

3) Integrate with respect to x:

a) $\frac{14-}{x^2+2x-8}$ b) $\frac{3x^2-5}{x^2-1}$ c) $\frac{x(4x+13)}{(2+x)^2(3-x)}$ **[23]**

4) Find the **exact** value of:

a) $\int_1^3 \frac{x+3}{x(x+1)}\,dx$ b) $\int_0^1 \frac{5x+7}{(x+1)^2(x+3)}\,dx$ **[15]**

5)

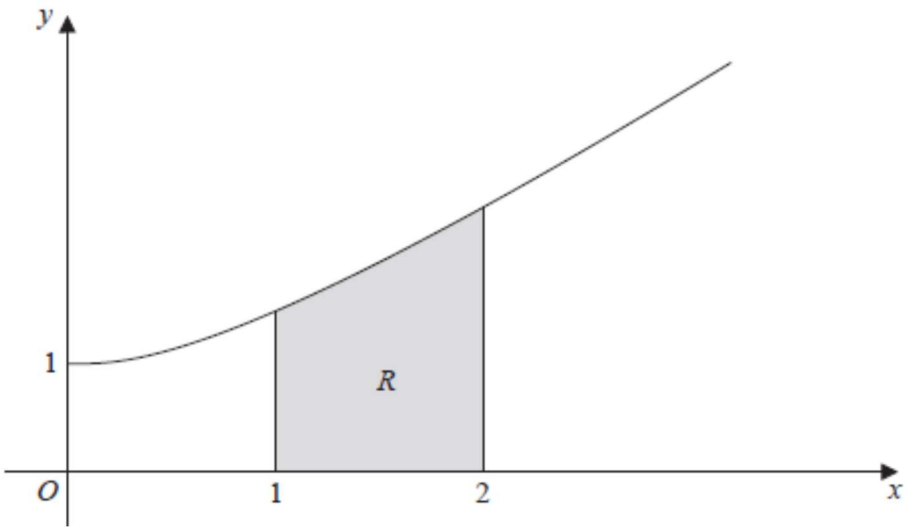

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}, x \geq 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$.

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414		1.803	2.016	2.236

a) Complete the table above, giving the missing value of y to 3 decimal places.

[1]

b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

[4]

6) The curve *C* has equation

$$y = 8 - 2^{x-1}, \quad 0 \leq x \leq 4.$$

a) Complete the table below with the value of *y* corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

b) Use the trapezium rule, with all the values of *y* in the completed table, to find an approximate value for $\int_0^4 (8 - 2^{x-1})\ \mathrm{d}x$.

(3)

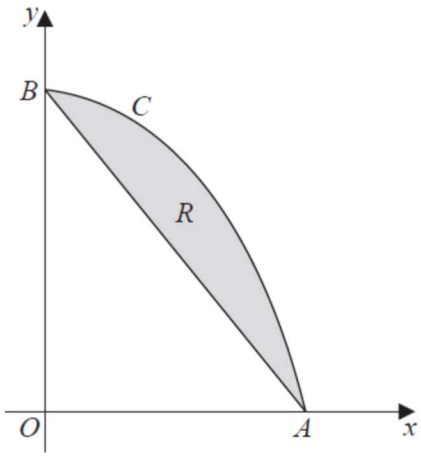

Figure 2

Figure 2 shows a sketch of the curve C with equation $= 8 - 2^{x-1}$, $0 \leq x \leq 4$.

The curve C meets the *x*-axis at the point A and meets the *y*-axis at the point B.

The region R, shown shaded in Figure 2, is bounded by the curve C and the straight line through A and B.

c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

7)

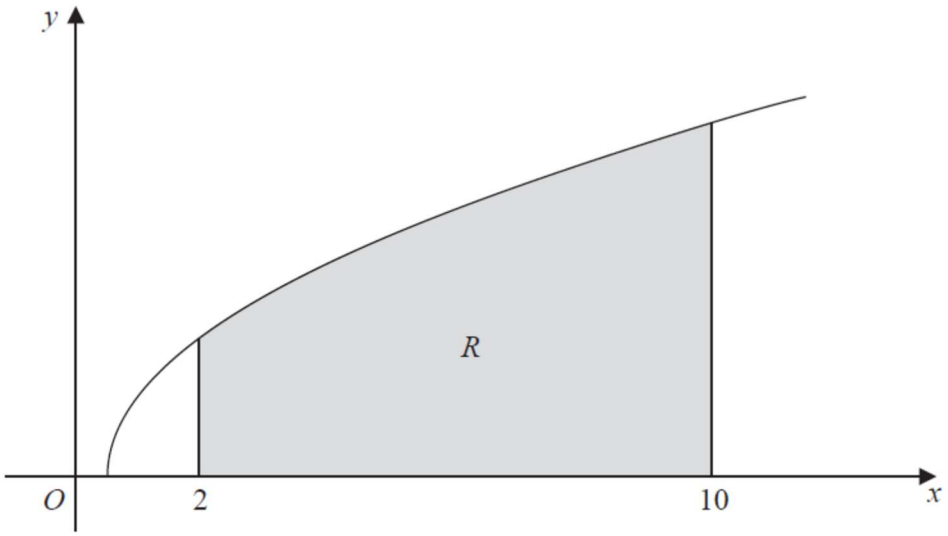

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \sqrt{(2x - 1)}, \ x \geq 0.5$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the *x*-axis and the lines with equations $x = 2$ and $x = 10$.

The table below shows corresponding values of x and y for $y = \sqrt{(2x - 1)}$.

x	2	4	6	8	10
y	√3		√11		√19

a) Complete the table with the values of y corresponding to $x = 4$ and $x = 8$.

[1]

b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

[3]

c) State, giving a reason, whether your approximate value in part (b) is an overestimate or an underestimate for the area of R.

[2]

Total: 69 Marks