Godalming College

Pure 48 – Differential Equations and Modelling

Please **complete** this homework by __________. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

Section 1 – Review of previous topics.
Please complete all questions.

1) a) Show that $(x-2)$ is a factor of $2x^3+x^2-7x-6=0$

b) Show that the equation $2x^3+x^2-7x-6$ has the solutions $2, -\frac{3}{2},$ and -1

2) Solve this equation, you must show your working:

$$4^x = 18 - 7(2^x)$$

3) The sum of the first two terms of an arithmetic series is 2. The sum of the first ten terms of the series is 330.

a) Work out the common difference of the series.

b) Write down the first term of the series

c) Given that the sum of the first n terms of the series is equal to 1170, find the value of n.

4) a) By writing $\cos 3x$ as $\cos(2x+x)$, show that $\cos 3x = 4\cos^3 x - 3\cos x$

b) Hence solve the equation $8\cos^3 x - 6\cos x = \sqrt{3}$ for x in the interval $0 \le x \le 2\pi$

5) a) Differentiate the following expressions with respect to x:

i) $5\cos x$ ii) $\ln x$ iii) $\frac{1}{4}e^x$

b) Find the equation of the normal to $y = 2\ln x$ at the point where $x = 1$. Give your answer in the form $ax + by + c = 0$ where a, b, and c are integers.

6) Prove that the derivative of $\arcsin 2x$ is $\frac{2}{\sqrt{1-4x^2}}$

Section 2 – Consolidation of this week's topic.
Please complete all questions.

1) Find the general solution to each of the following differential equations:

a) $\frac{dy}{dx} - 2x + 1 = 0$ b) $\frac{dy}{dx} = \frac{x}{y}$ c) $\frac{dy}{dx} = 5xy$ d) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$ **(14)**

2) Find the particular solution to the differential equation that corresponds to the given initial conditions:

a) $\frac{dy}{dx} + 9x^2 - 2 = 0$; $(2, -10)$ b) $\frac{dy}{dx} = x(y+3)$; $(2, -2)$ c) $2x\frac{dy}{dx} - y = 0$ $(1, 2)$ **(13)**

3) Express each sentence as a differential equation:

a) A ball rolls down an inclined plane. The rate at which its distance from the top changes with respect to time is directly proportional to the time it has been rolling. Let S cm be the distance after t seconds.

b) When selling luxury items the rate of change of demand, D, with respect to the price, P, is inversely proportional to the price.

c) Earthquakes are measured using the Richter Scale. The rate of change of an earthquake's intensity, as its Richter scale number changes, is directly proportional to 10^R. R is the Richter scale number when the intensity is I units. **(3)**

4) Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

(5)

5) The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{dx}{dt} = -\frac{5}{2}x, \qquad t \geq 0$$

where x is the mass of the substance measured in grams and t is the time measured in days. Given that $x = 60$ when $t = 0$,

a) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

b) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams. Give your answer to the nearest minute.

(3)

6) Over a month, the rate at which the percentage of the moon which is visible changes with time can be modelled by $\frac{dP}{dt} = \frac{25\pi}{7}\cos\left(\frac{\pi}{14}t\right)$ where P is the percentage visible on day t of the month.

a) Solve the differential equation to express P in terms of t and a constant

b) The month began with a half moon. Express P in terms of t.

c) What percentage of the moon was visible on day 4?

d) On which day was there a full moon? **(9)**

7) An osprey can be expected to reach an adult weigh of 2000 g.

On day zero, a chick will weigh 50g on hatching. It fledges after 60 days when its weight is 1990 g.

Its rate of growth is directly proportional to the difference between its weight and its expected adult weight. On day t, its weight is w grams.

a) Form a differential equation to model the development of the osprey chick.

b) Evaluate the constant of integration if at $t = 0$, $w = 50$

c) Find the constant of proportion

d) Express w explicitly as a function of t

e) When is the chick's weight expected to exceed 1500 g?

f) Discuss any assumptions and limitations of the model. **(14)**

Total: 65 Marks