Pure 49 – Parametric Integration

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop-in session.

Section 1 – Review of previous topics. Please complete all questions.

1. Given that $\frac{dx}{dt} = (2t-1)^2$ and that $x = 0$ when $t = 3$ find the value of x when $t = 6$

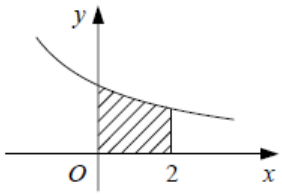
2. A motorbike is travelling along a straight road. The distance in meters of the motorbike from a fixed point after t seconds is modelled by the function $f(t)$, where $f'(t) = 6 + 5t$ and f(0)=0.

   a) Find an expression for $f(t)$

   b) Calculate the time taken for the motorbike to travel 200m

Section 2 – Consolidation of this week's topic. Please complete all questions.

1.



The diagram shows part of the curve with parametric equations $x = 2t - 4, y = \frac{1}{t}$.

The shaded region is bounded by the curve, the coordinate axes and the line $x = 2$

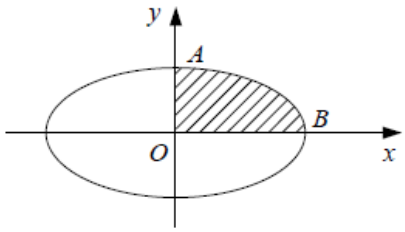
a) Find the value of the parameter t when $x = 0$ and when $x = 2$. (3 marks)

b) Show that the area of the shaded region is given by $\int_2^3 \frac{2}{t}\, dt$. (4 marks)

c) Hence, find the area of the shaded region. (3 marks)

d) Verify your answer to part c) by first finding the cartesian equation for the curve (4 marks)

2.



The diagram shows the ellipse with parametric equations $x = 4cos\theta, y = 2sin\theta$, $0 \leq \theta < 2\pi$,

which meets the positive coordinate axes at the points A and B.

a) Find the value of the parameter θ at the points A and B (2 marks)

b) Show that the area of the shaded region bounded by the curve and the positive coordinate axes is given by $\int_0^{\frac{\pi}{2}} 8sin^2\theta \, d\theta.$ (4 marks)

c) Hence, show that the area of the region enclosed by the ellipse is 8π (4 marks)

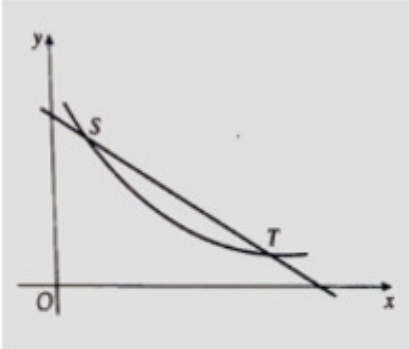
3. The diagram shows a sketch of the curve with parametric equations $x = at, y = \frac{4a}{t}, t > 0$, and the line $y = 5a - x$, where a is a constant.

The line meets the curve at S and T.

a) Find, in terms of a, the co-ordinates of the points S and T (5 marks)

b) Show that $\int y \frac{dx}{dt} \, dt = 4a^2 lnt + c$, where c is a constant. (2 marks)

c) Hence find, in terms of a, the exact area of the finite region between the curve and the line. (5 marks)



Total: 36 marks