Pure 1 – Quadratics, Solutions

Section 1

a) $x^3 - 1$ b) $2x^2 + 6x^3$ c) $3 - x^2$ d) $12x^3 + 8x$
e) $3x^3 + 2x$ f) $3 - 3x^2$ g) $5x^{1/2} + x^2$ h) $3x^2 - x^{-1}$
i) $x^6 + x^4 - 3x^2 - 3$ j) $2x^9 + 7x^5 + 3x$ k) $x^3 - 3 + 2x^{-3}$ l) $x^3 - 2x^{3/2} + x^2$

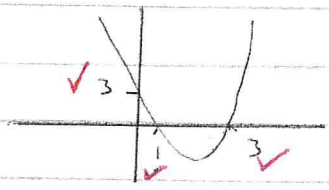
a) $1/4$ b) 1 c) $1/64$ d) 36 e) $\frac{8}{27}$ f) 3
g) 2 h) -3 i) $1/7$ j) $\frac{1}{5}$ 5 k) $2/3$ l) $1/6$
m) $1/3$ n) $-1/4$ o) 2 p) $-2/5$ q) $3/2$ r) $2/3$

a) 7 b) 20 c) 27 d) 36
e) $4\sqrt{2}$ f) $24\sqrt{3}$ g) 4 h) 18
i) 4 j) $1/2$ k) 6 l) 54

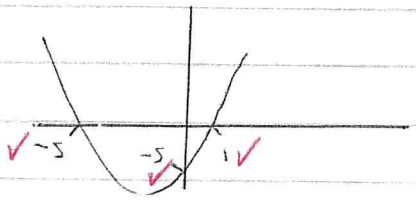
Section 2:

Total marks (125)
5 marks each

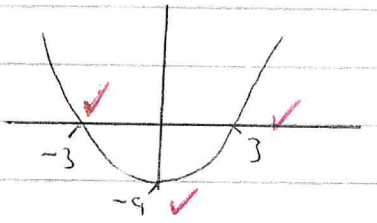
1 (a) $y = (x-3)(x-1)$
$y = 0$, $x = 1, 3$
$x = 0$, $y = 3$



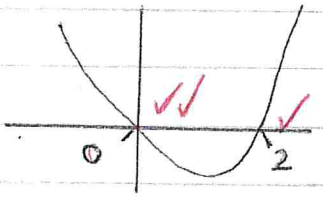
(b) $y = (x+5)(x-1)$
$x = -5$ and 1 when $y = 0$
$y = -5$ when $x = 0$



(c) $y = (x+3)(x-3)$ (difference of two squares)
$x = 3$ and -3 when $y = 0$
$y = -9$ when $x = 0$



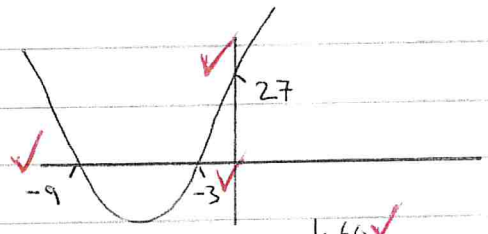
(d) $y = x(x-2)$
$x = 0$ and 2 when $y = 0$
$y = 0$ when $x = 0$



e) $y = (x+3)(x+9)$

$x = -3, -9$

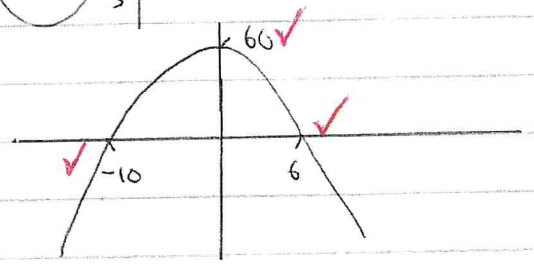
y intercept is 27



f) $y = (-x+6)(x+10)$

$x = 6, -10$

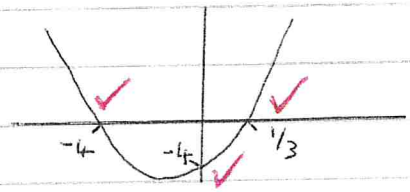
y intercept 60



g) $y = (3x-1)(x+4)$

x intercepts $1/3, -4$

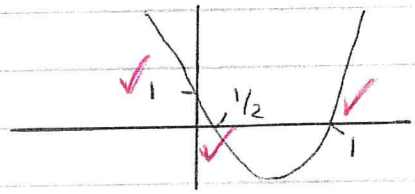
y intercept -4



h) $y = (2x-1)(x-1)$

x intercepts $1/2, 1$

y intercept 1



(40)

2 (a) $y = (x-2)^2 + 2 - 4$

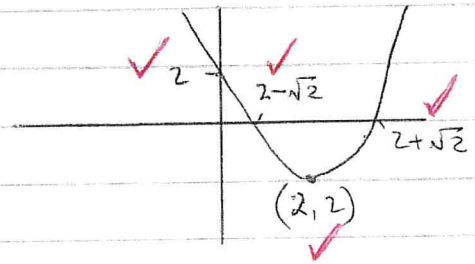
$y = (x-2)^2 - 2 = 0$

$(x-2)^2 = 2$

$x = 2 \pm \sqrt{2}$

y intercept is $+2$

minimum at $(2, 2)$



(6 each)

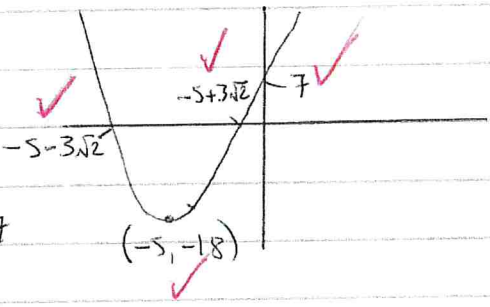
(b) $y = (x+5)^2 + 7 - 25$

$y = (x+5)^2 - 18 = 0$

$(x+5)^2 = 18 \Rightarrow x = -5 \pm \sqrt{18}$

$x = -5 \pm 3\sqrt{2}$, y intercept at 7

minimum at $(-5, -18)$

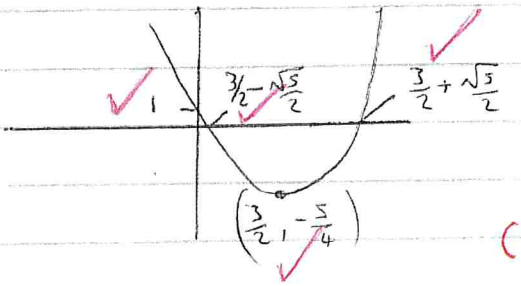


(c) $y = (x - \frac{3}{2})^2 + 1 - \frac{9}{4}$

$y = (x - \frac{3}{2})^2 - \frac{5}{4} = 0$

$x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$, y intercept at 1

minimum at $(\frac{3}{2}, -\frac{5}{4})$



(18)

Section 2

(5 each)

2. (d) $y = 2[x^2 - 2x] + 1$

$y = 2[(x-1)^2 - 1] + 1$ ✓

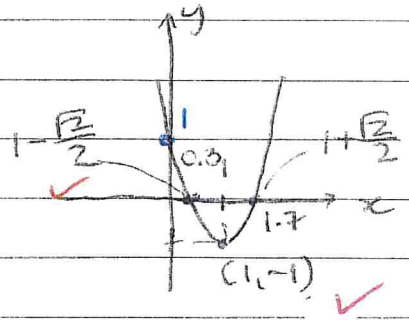
$y = 2(x-1)^2 - 1$

min $(1, -1)$ ✓

$2(x-1)^2 = 1$

$(x-1)^2 = \frac{1}{2}$

$x = 1 \pm \frac{\sqrt{2}}{2}$ $\quad x = 1.7$ or $x = 0.3$



✓ ✓ ✓

y intercept

$y = 1$

(e) $y = -[x^2 - x] + 1$

$y = -[(x-0.5)^2 - 0.25] + 1$

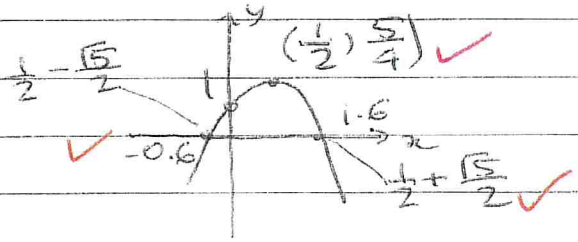
$y = -(x-0.5)^2 + 1.25$ ✓

max $(0.5, 1.25)$

$(x-0.5)^2 = 1.25$

$x = 0.5 \pm \sqrt{1.25}$ ✓

or $x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$



✓ ✓ ✓

y intercept $y = 1$

$x = 1.6$ or -0.6

(f) $y = 2\left[x^2 - \frac{7}{2}x\right] + 4$

$y = 2\left[(x-\frac{7}{4})^2 - \frac{49}{16}\right] + 4$

$y = 2(x-\frac{7}{4})^2 - \frac{49}{8} + 4$ ✓

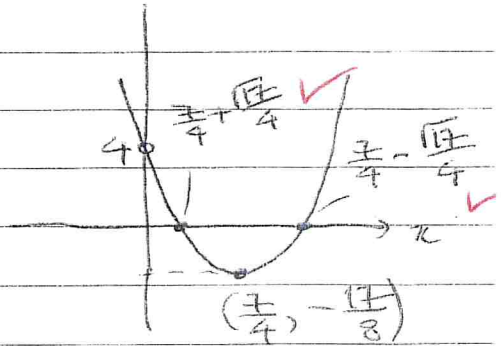
$y = 2(x-\frac{7}{4})^2 - \frac{17}{8}$

min $(\frac{7}{4}, -\frac{17}{8})$

$2(x-\frac{7}{4})^2 = \frac{17}{8}$ ÷2 ✓

$(x-\frac{7}{4})^2 = \frac{17}{16}$

$x = \frac{7}{4} \pm \frac{\sqrt{17}}{4}$



✓ ✓ ✓

y intercept

$y = 4$

(15)

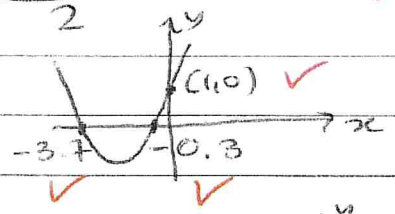
Q2 : 15+18 = (33)

Section 2

(5) each

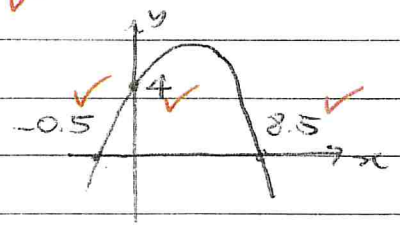
3. (a) $x = \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$ ✓

$x = -3.7$ or -0.3

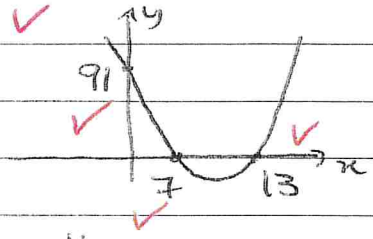


(b) $x = \frac{-8 \pm \sqrt{64+16}}{-2}$ ✓

$x = \frac{-8 \pm 4\sqrt{5}}{-2} = 4 \pm 2\sqrt{5}$ ✓ $x = 8.5$ $x = -0.5$

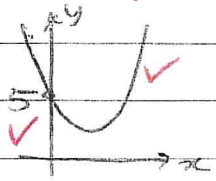


(c) $x = \frac{20 \pm \sqrt{400-364}}{2} = \frac{20 \pm 6}{2} \Rightarrow x = 7$ or 13 ✓



(d) $x = \frac{7 \pm \sqrt{49-60}}{6} = \frac{7 \pm \sqrt{-11}}{6}$ ✓

NO SOL^N.



(20)

4. (a) $b^2 - 4ac = 32$ ∴ real and distinct (2) each

(b) -11 ∴ not real

(c) 24 ∴ real and distinct

(d) -7 ∴ not real

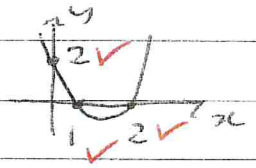
(e) -11 ∴ not real. (f) 16 ∴ real and distinct (12)

5. (a) $(x-2)(x-1) = 0$

$x = 2$ $x = 1$

$y = 2$

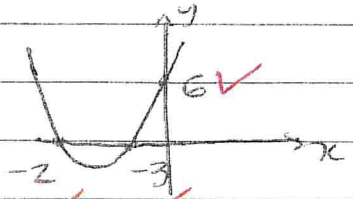
(5) each



(b) $(x+2)(x+3) = 0$

$x = -2$ $x = -3$

$y = 6$

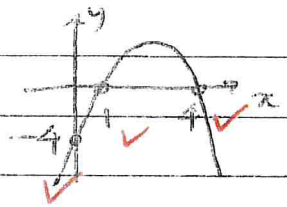


5. (c) $y = -x^2 + 5x - 4$

$-(x-4)(x-1) = 0$ ✓

$x = 4 \quad x = 1$ ✓

$y = -4$ ✓



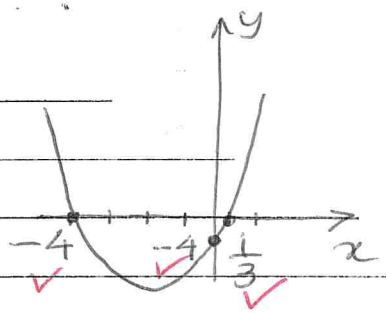
✓ ✓ ✓

(d) $y = 3x^2 + 11x - 4$

$3x^2 + 11x - 4 = 0$

$(3x-1)(x+4) = 0$ ✓

$x = \frac{1}{3} \quad x = -4$ ✓



✓ ✓ ✓

(20)

total: (125)

Section 3

1.

$$ax^2+bx+c=0$$

$$a\left[x^2+\frac{b}{a}x\right]+c=0$$

$$a\left[\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2\right]+c=0 \quad \div a$$

$$\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right]+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a} \quad \times 4a$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2} \quad \text{take } \sqrt{}$$

$$x+\frac{b}{2a}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ //

2. $y=x^2+rx-2x+4$ x-axis is a tangent $\Rightarrow y=0$

$\Rightarrow x^2+rx-2x+4=0$

$x^2+(r-2)x+4=0 \quad a=1,\ b=r-2,\ c=4$

$$x=\frac{-(r-2)\pm\sqrt{(r-2)^2-4(4)}}{2}$$

To be a tangent $\therefore$ discriminant $=0$

$(r-2)^2-16=0 \quad (r-2)^2=16 \quad r-2=\pm4$ or

$r=2+4=6$ // $\quad r=2-4$

$r=-2$ //