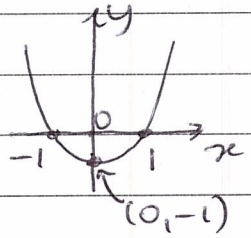
Pure 2 – Equations and inequalities
Solutions.

Section 1

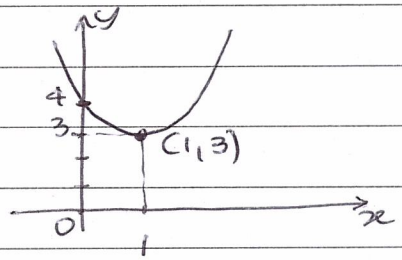
1. (a) $0, 5$ (b) $-3, \frac{5}{2}$ (c) $1 \pm \frac{\sqrt{2}}{2}$

2. (a) $-13 < 0 \therefore$ no sol (b) $12 > 0 \therefore$ 2 sols or 2 distinct real roots. (c) 0, one repeated root

3. (a)



(b)



Section 2

1. (a) $\left.\begin{matrix} 2x+3y=-7 \\ 5x-2y=11 \end{matrix}\right\} \begin{matrix} \times 2 \Rightarrow \\ \times 3 \Rightarrow \end{matrix} \left.\begin{matrix} 4x+6y=-14 \\ 15x-6y=33 \end{matrix}\right\}$ ✓

Add $19x = 19 \rightarrow x = 1$ ✓

subst $x=1$ in $2x+3y=-7$

$2(1)+3y=-7$ ✓

$3y=-9 \quad y=-3$ ✓ $\underline{(1,-3)}$ ✓ (5)

(b) $\left.\begin{matrix} y=x-3 \\ y^2+xy+4x=7 \end{matrix}\right\} \begin{matrix} -① \\ -② \end{matrix}$ → rearrange $x=y+3$

will be easier, no x^2.

subst for x in ②

$\Rightarrow y^2+(y+3)y+4(y+3)=7$ ✓

$y^2+y^2+3y+4y+12=7$ ✓

$2y^2+7y+5=0$

$(2y+5)(y+1)=0$ ✓ $\rightarrow y=-\frac{5}{2}$ or -1 ✓

when $y=-\frac{5}{2}$, $x=-\frac{5}{2}+3=\frac{1}{2}$

when $y=-1$, $x=-1+3=2$

solutions are $\underline{\left(\frac{1}{2}, -\frac{5}{2}\right) \text{ and } (2,-1)}$ ✓ (5)

(c) $x+2y=13$ } -(1) $x=13-2y$ ✓
$x^2-y^2=9$ } -(2)

subst for x in (2) $\Rightarrow$

take care expanding → $(13-2y)^2-y^2=9$ ✓

$169-52y+4y^2-y^2=9$

$169-52y+3y^2=9$

$3y^2-52y+160=0$

$(3y-40)(y-4)=0$ ✓

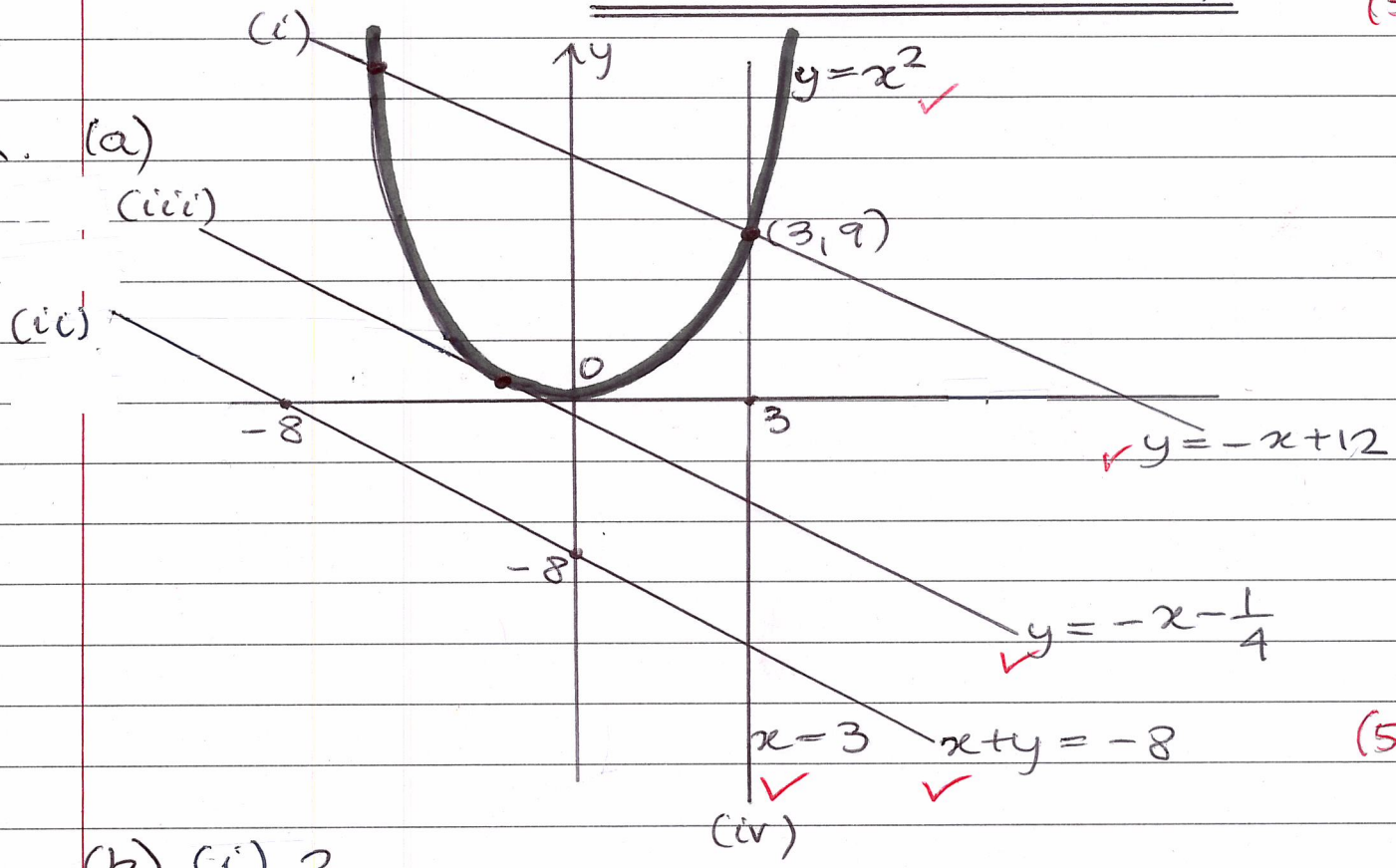
$y=4$ or $\frac{40}{3}$ ✓

when $y=4$, $x=13-8=5$

when $y=\frac{40}{3}$, $x=13-\frac{80}{3}=\frac{39-80}{3}=-\frac{41}{3}$

solutions are $(5,4)$ and $\left(-\frac{41}{3},\frac{40}{3}\right)$ ✓ (5)

2. (a)



(5)

(b) (i) 2

(ii) 0

(iii) 1 repeated root. The line is a tangent.

(iv) 1 Not a repeated root, the line is not a tangent.

(4)

2. (c)

(i) $y = x^2$
$y = -x + 12$

$x^2 = -x + 12$
$x^2 + x - 12 = 0$ ✓
$(x-3)(x+4) = 0$
$x = 3$ or $x = -4$ ✓ ✓

when $x = 3$ $y = 9$
$x = -4$ $y = 16$.

Sol: $(3, 9)$ and $(-4, 16)$ ✓ ✓ (5)

(ii) $y = x^2$
$x + y = -8$

$y = x^2$
$y = -8 - x$
$x^2 = -8 - x$
$x^2 + x + 8 = 0$ ✓

Use the quadratic formula $a = 1$ $b = 1$ $c = 8$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 32}}{2}$ ✓

$x = \frac{-1 \pm \sqrt{-31}}{2}$ ✓ No real roots ie no solutions. ✓ (4)

(iii) $y = x^2$
$y = -x - \frac{1}{4}$

$x^2 = -x - \frac{1}{4}$ $\times 4$
$4x^2 = -4x - 1$
$4x^2 + 4x + 1 = 0$ ✓
$(2x+1)(2x+1) = 0$ ✓
$x = -\frac{1}{2}$ ✓

when $x = -\frac{1}{2}$
$y = \frac{1}{4}$

One repeated root ie one solution $(-\frac{1}{2}, \frac{1}{4})$ ✓ (4)

(iv) $y = x^2$
$x = 3$

when $x = 3$ $y = 3^2 = 9$ ✓
One point of intersection $(3, 9)$. ✓ (2)

3. (a) $3(y-1) \geq 5y-8$

$3y-3 \geq 5y-8$ ✓

$5 \geq 2y$ ✓

$y \leq 2.5$ ✓ (3)

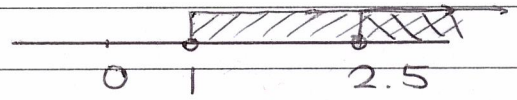
(b) $3(x-2) > x-4$ and $4x+12 > 2x+17$

$3x-6 > x-4$ $\quad 2x > 5 \div 2$

$2x > 2$ $\quad x > 2.5$ ✓

$x > 1$ ✓

sol:



$x > 2.5$ ✓ (3)

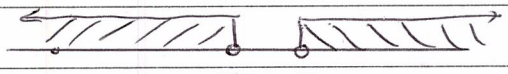
(c) $15-x < 2(11-x)$ and $5(3x-1) > 12x+19$

$15-x < 22-2x$ $\quad 15x-5 > 12x+19$

$x < 7$ ✓ $\quad 3x > 24$

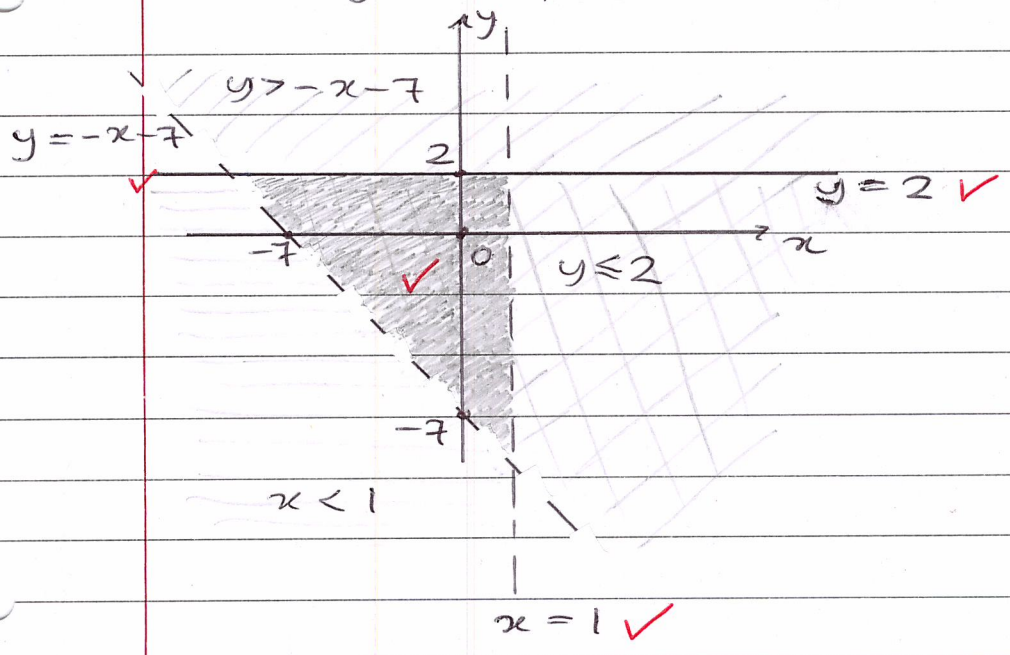
$x > 8$ ✓

sol:



No solutions ✓ (3)

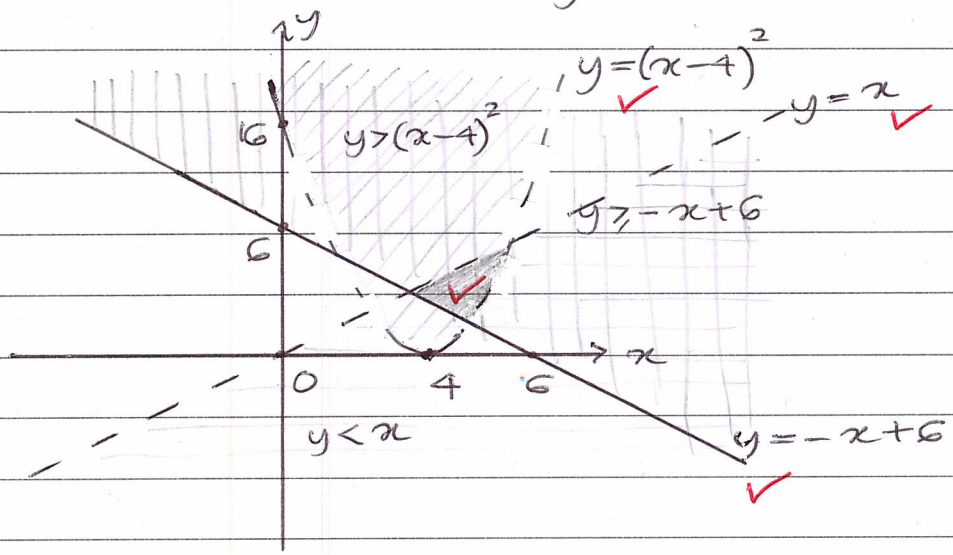
4. (a) $x+y > -7$, $y \leq 2$ and $x < 1$.



(4)

4. (b) $y > (x-4)^2$, $y+x \geq 6$ and $y < x$

$y \geq -x+6$



(4)

Section 3:

1. $P(-1,6)$ $Q(3,2)$ $PQ = 4\sqrt{2}$

2. $x = 3$, $y = \frac{1}{2}$

TOTAL: (56)