Pure 3 – Inequalities Solutions

Section 1:

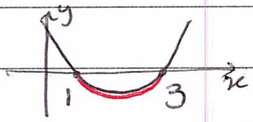
① (a) $\frac{1}{2} \le y < 7$ (b) $x > 3$ (c) $-23 < x < -\frac{1}{2}$

② $(-2,-1)$ and $\left(\frac{11}{5}, \frac{2}{5}\right)$.

Section 2

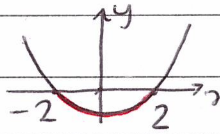
3 marks each:

① (a) $(x-1)(x-3) < 0$



$\underline{\underline{1 < x < 3}}$

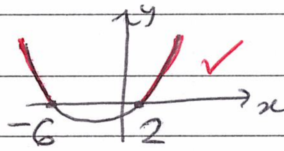
(b) $(x+2)(x-2) \le 0$



$\underline{\underline{-2 \le x \le 2}}$

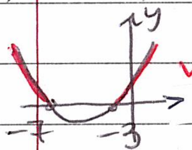
(c) $x^2+4x-12 > 0$

$(x+6)(x-2) > 0$



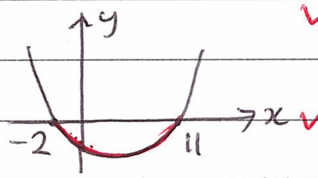
$\underline{\underline{x < -6 \text{ or } x > 2}}$

(d) $(x+7)(x+3) \ge 0$



$\underline{\underline{x \le -7 \text{ or } x \ge -3}}$

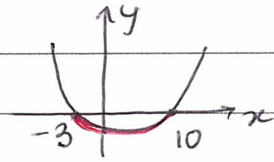
(e) $(x+2)(x-11) < 0$



$\underline{\underline{-2 < x < 11}}$

(f) $x^2-7x-30 < 0$

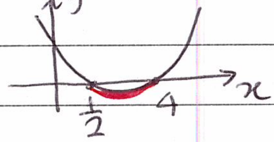
$(x+3)(x-10) < 0$



$\underline{\underline{-3 < x < 10}}$.

(18)

② (a) $(2x-1)(x-4) \le 0$

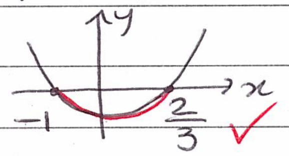


$\underline{\underline{\frac{1}{2} \le x \le 4}}$

(b) $3p^2+p-2 \le 0$

3 each:

$(3p-2)(p+1) \le 0$

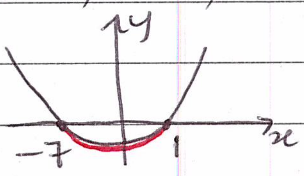


$\underline{\underline{-1 \le p \le \frac{2}{3}}}$

(c) $x^2+4x \le 7-2x$

$x^2+6x-7 \le 0$

$(x+7)(x-1) \le 0$

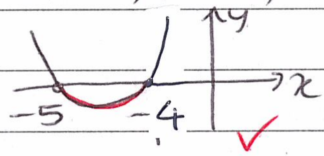


$\underline{\underline{-7 \le x \le 1}}$

(d) $26+4x < 6-5x-x^2$

$x^2+9x+20 < 0$

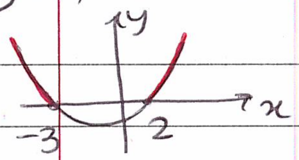
$(x+5)(x+4) < 0$



$\underline{\underline{-5 < x < -4}}$

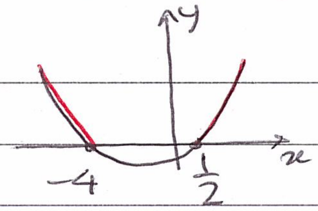
(12)

(2) (e) $(x+3)(x-2) > 0$



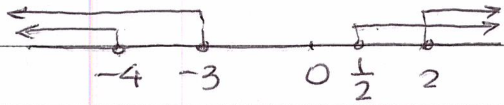
$x < -3$ or $x > 2$

$(2x-1)(x+4) > 0$



$x < -4$ or $x > \frac{1}{2}$

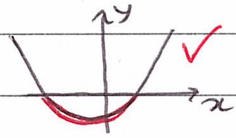
solution:



$\underline{\underline{x < -4 \text{ or } x > 2}}$ (5)

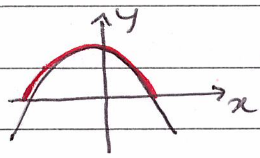
(3) (a) for critical values

$$x = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = \frac{-2}{2} \pm \frac{2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$



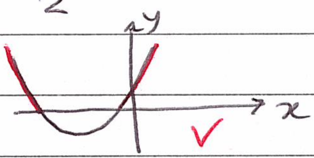
$\underline{\underline{-1-\sqrt{2} < x < -1+\sqrt{2}}}$

(b) $$x = \frac{6 \pm \sqrt{36+44}}{-2} = \frac{6 \pm 4\sqrt{5}}{-2} = \frac{6}{-2} \pm \frac{4\sqrt{5}}{-2} = -3 \pm 2\sqrt{5}$$



$\underline{\underline{-3-2\sqrt{5} < x < -3+2\sqrt{5}}}$

(c) $$x = \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = \frac{-4}{2} \pm \frac{2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$



$\underline{\underline{x \leq -2-\sqrt{3} \text{ or } x \geq -2+\sqrt{3}}}$

(9)

(4) (a) equal roots $b^2 - 4ac = 0$ — 2 marks each:

$36 - 4k = 0 \Rightarrow \underline{\underline{k = 9}}$

(b) real and distinct roots $\therefore$ $b^2 - 4ac > 0$

$4 - 4k > 0$

$4 > 4k \Rightarrow \underline{\underline{k < 1}}$

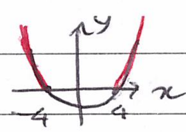
(c) no real roots $\therefore$ $b^2 - 4ac < 0$

$9 - 4k < 0$

$9 < 4k \Rightarrow \underline{\underline{k > \frac{9}{4}}}$

(d) real roots:

$b^2 - 4ac \geq 0 \therefore k^2 - 16 \geq 0$, $(k+4)(k-4) \geq 0$



$\underline{\underline{k \leq -4 \text{ or } k \geq 4}}$

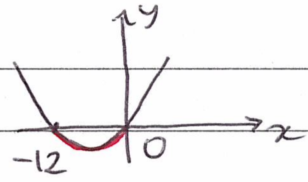
(4)

(e) no real roots $\therefore\ b^2-4ac<0$

$k^2+12k<0$ ✓

$k(k+12)<0$

$\underline{\underline{-12<k<0}}$ ✓



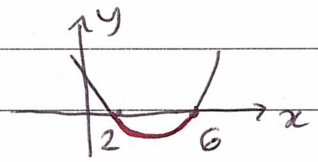
(f) no real roots:

$\therefore\ b^2-4ac<0$

$k^2-4(2k-3)<0$ ✓

$k^2-8k+12<0$

$(k-2)(k-6)<0 \Rightarrow \underline{\underline{2<k<6}}$ ✓



(g) real and distinct roots

$b^2-4ac>0 \ \therefore\ 4-4(k-2)>0$ ✓ $\Rightarrow 12>4k \quad \underline{\underline{k<3}}$ ✓

(h) equal roots $\therefore\ b^2-4ac=0$

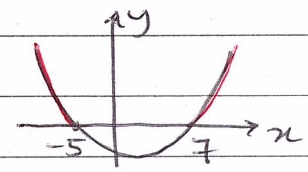
$k^2-8k=0$ ✓ $\Rightarrow k(k-8)=0 \quad \underline{\underline{k=0 \text{ or } 8}}$ ✓

(i) real roots $\therefore\ b^2-4ac\geq 0$

$(k-1)^2-36\geq 0$

$k^2-2k-35\geq 0$ ✓

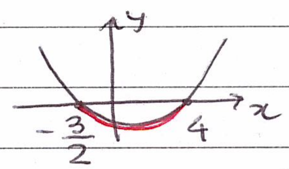
$(k+5)(k-7)\geq 0$



$\underline{\underline{k\leq -5 \text{ or } k\geq 7}}$ ✓ (18)

(5) $2n^2-5n-12<0$

$(2n+3)(n-4)<0$ ✓



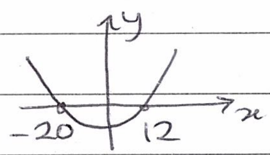
$-\frac{3}{2}<n<4$ ✓ (4)

$\underline{\underline{n \text{ integer}: -1, 0, 1, 2, 3}}$ ✓

(6) $x=y+8 \Rightarrow y(y+8)\leq 240$ ✓ $\Rightarrow y^2+8y-240\leq 0$

$(y+20)(y-12)\leq 0$ ✓

$-20\leq y\leq 12$ ✓



$x+y=y+8+y=2y+8$

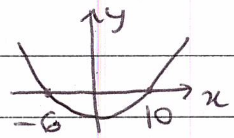
$\therefore$ max value of $\underline{\underline{(x+y)=2(12)+8=32}}$ ✓ (4)

Total (70)

Section 3

1. (a) set $b^2 - 4ac > 0$ as real and distinct roots.

(b) $(k+6)(k-10) > 0$



$k < -6$ or $k > 10$

2. Let height be h $\therefore$ $h^2 = (3r-4)^2 - r^2$

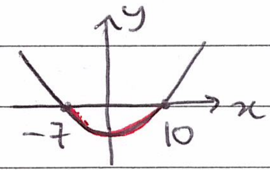
but $h \leq 24$

$h^2 \leq 24^2$

$(3r-4)^2 - r^2 \leq 576$

$r^2 - 3r - 70 \leq 0$

$(r+7)(r-10) \leq 0$



$-7 \leq r \leq 10$

$\therefore$ max value of

$r = 10$