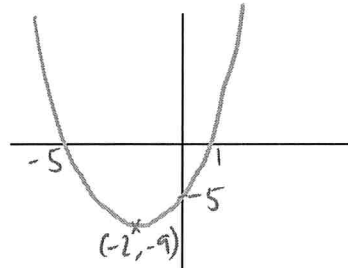


# Pure 6 – Equation of a Circle

## Section 1

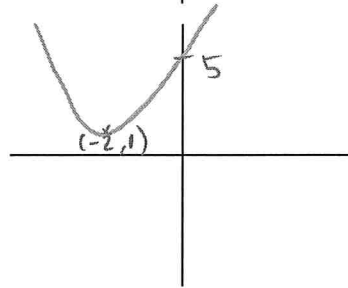
- ① i) a) 2 distinct real roots  
 b)  $x = 1, -5$   
 c)  $(x+2)^2 - 9 = y$   
 Min:  $(-2, -9)$

d)



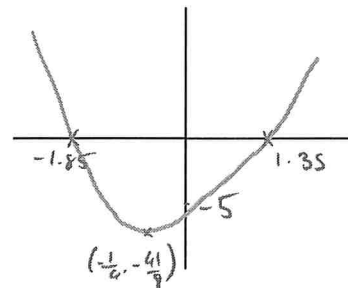
- ii) a) no real roots  
 b) n/a  
 c)  $(x+2)^2 + 1 = y$   
 Min:  $(-2, 1)$

d)



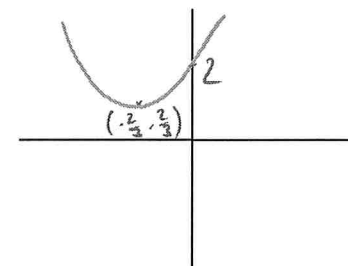
- iii) a) 2 distinct real roots  
 b)  $x = \frac{-1 \pm \sqrt{41}}{4}$   
 c)  $2(x + \frac{1}{4})^2 - \frac{41}{8} = y$   
 Min:  $(-\frac{1}{4}, -\frac{41}{8})$

d)



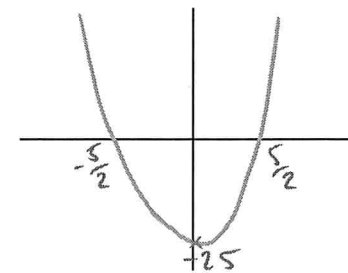
- iv) a) no real roots  
 b) n/a  
 c)  $3(x + \frac{2}{3})^2 + \frac{2}{3} = y$   
 Min:  $(-\frac{2}{3}, \frac{2}{3})$

d)



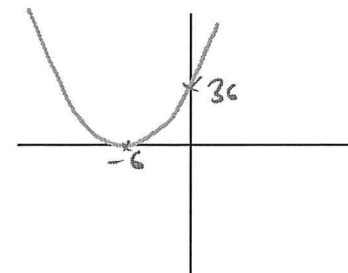
- v) a) 2 distinct real roots  
 b)  $x = \frac{5}{2}, -\frac{5}{2}$   
 c)  $4x^2 - 25 = y$   
 Min:  $(0, -25)$

d)



- vi) a) 1 repeated root  
 b)  $x = -6$   
 c)  $(x-6)^2 = y$   
 Min:  $(-6, 0)$

d)



$$2a) \text{ midpoint } \overline{BC} = \left( \frac{2+9}{2}, \frac{9+2}{2} \right) = (5.5, 5.5)$$

$$\text{gradient } \overline{BC} = \frac{2-9}{9-2} = \frac{-7}{7} = -1 \quad \therefore \text{perpendicular gradient} = 1$$

$$y - 5.5 = 1(x - 5.5)$$

$$y = x$$

$$b) \text{ gradient } \overline{AB} = \left( \frac{9-(-2)}{2-5} \right) = -\frac{11}{3}$$

$$\text{sub in } y = x$$

$$11x + 3x - 49 = 0$$

$$14x = 49$$

$$x = 3.5$$

$$(3.5, 3.5)$$

$$y - (-2) = -\frac{11}{3}(x + 5)$$

$$y + 2 = -\frac{11}{3}x + \frac{55}{3}$$

$$3y + 6 = -11x + 55$$

$$11x + 3y - 49 = 0$$

$$3a) \left( \frac{h+3h}{2}, \frac{k+(-5k)}{2} \right) = \left( \frac{4h}{2}, \frac{-4k}{2} \right) \\ = (2h, -2k)$$

$$b) \frac{k - (-5k)}{h - 3h} = -\frac{6k}{2h} = -\frac{3k}{h}$$

$$c) \text{ perpendicular gradient} = \frac{h}{3k}$$

$$y - (-2k) = \frac{h}{3k}(x - 2h)$$

$$y + 2k = \frac{hx}{3k} - \frac{2h^2}{3k}$$

$$3ky + 6k^2 = hx - 2h^2$$

$$hx - 3ky - 6k^2 - 2h^2 = 0$$

## Section 2 (Pure 6 Equation of a Circle)

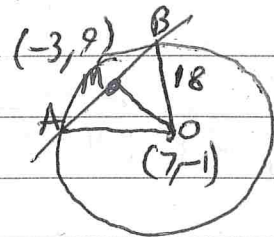
1) a)  $(x-1)^2 + (y-8)^2 = 5^2 \Rightarrow x^2 - 2x + y^2 - 16y + 40 = 0$

8) b)  $(x-6)^2 + (y+7)^2 = 3^2 \Rightarrow x^2 - 12x + y^2 + 14y + 76 = 0$

2) a)  $(x+9)^2 + (y-7)^2 = 100$  Centre  $(-9, 7)$  radius = 10

8) b)  $(x+6)^2 + (y+5)^2 = 86$  Centre  $(-6, -5)$  radius =  $\sqrt{86}$

3) a)  $(x-7)^2 + (y+1)^2 = 18^2$



b)  $OM = \sqrt{(7+3)^2 + (-1-9)^2} = \sqrt{200}$

$\Rightarrow MB^2 = 18^2 - 200 = 124$

$\Rightarrow AB = 2\sqrt{124} = 4\sqrt{31}$

c) gradient  $OM = \frac{10}{-10} = -1$  gradient  $AB = 1$

$y-9 = 1(x+3)$   $y = x+12$  meets circle

where  $(x-7)^2 + (x+13)^2 = 18^2$

$\Rightarrow 2x^2 + 12x - 106 = 0 \Rightarrow x^2 + 6x - 53 = 0$

$(x+3)^2 = 62 \Rightarrow x = -3 \pm \sqrt{62}$

$y = 9 \pm \sqrt{62}$

4) a) centre  $(4, 4)$  radius  $\frac{1}{2}\sqrt{4^2 + 4^2} = 2\sqrt{2}$

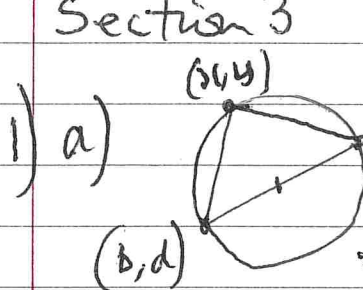
$(x-4)^2 + (y-4)^2 = 8 \Rightarrow x^2 - 8x + y^2 - 8y + 24 = 0$

8) b) centre  $(\frac{1}{2}, 7)$  radius  $\frac{1}{2}\sqrt{7^2 + 18^2} = \frac{1}{2}\sqrt{373}$

$\Rightarrow x^2 + x + y^2 - 14y - 44 = 0$

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### Section 3

1) a)  (PYTHAG!)  
 $(a-b)^2 + (c-d)^2 = (x-b)^2 + (y-d)^2 + (x-a)^2 + (y-c)^2$   
 $\Rightarrow a^2 - 2ab + b^2 + c^2 - 2cd + d^2 = x^2 - 2bx + b^2 + y^2 - 2dy + d^2 + x^2 - 2ax + a^2 + y^2 - 2cy + c^2$   
 $\div 2 \Rightarrow -ab - cd = x^2 - bx - ax + y^2 - dy - cy$   
 $0 = x^2 - bx - ax + ab + y^2 - dy - cy + cd$   
 $0 = (x-a)(x-b) + (y-c)(y-d)$

2) a)  $(x-9)^2 + (y + \frac{k}{2})^2 = \frac{k^2}{4} - 9$  centre  $(9, -\frac{k}{2})$   
 radius  $\sqrt{\frac{k^2}{4} - 9}$

b)  $C_2: (x-1)^2 + (y-1)^2 = 36$

(Length between centres) =  $(r_1 + r_2)^2$

$(9-1)^2 + (-\frac{k}{2}-1)^2 = (6 + \sqrt{\frac{k^2}{4} - 9})^2$

$\Rightarrow 64 + \frac{k^2}{4} + k + 1 = 36 + 12\sqrt{\frac{k^2}{4} - 9} + \frac{k^2}{4} - 9$

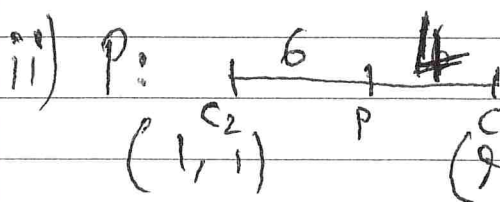
$\Rightarrow 64 + k + 1 = 27 + 12\sqrt{\frac{k^2}{4} - 9}$

$\Rightarrow k + 38 = 12\sqrt{\frac{k^2}{4} - 9}$  Square both sides

$\Rightarrow k^2 + 76k + 1444 = \frac{144k^2}{4} - 1296$

$\Rightarrow 35k^2 - 76k - 2740 = 0$

i) From Formula  $k = 10$  or  $-\frac{274}{35}$

ii) P:   $P = (1, 1) + \frac{6}{10}(8, -6)$   
 $(1, 1) \quad P \quad (9, -5) = (\frac{29}{5}, -\frac{13}{5})$