Pure 7 Problems on Circles

SECTION 1

1) a) $3\sqrt{3}+2\times5\sqrt{2}=10\sqrt{2}+3\sqrt{3}$ b) $\sqrt{18}-\sqrt{48}=3\sqrt{2}-4\sqrt{3}$

2) $6^{y+1}=(6)^{2(x-2)}$ a) $\Rightarrow y+1=2x-4 \Rightarrow \underline{y=2x-5}$

b) $4^{x-\frac{1}{2}(2x-5)}=4^{5/2}=2^5=32$

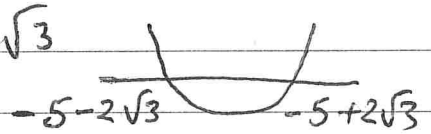
3) a) $(x-6)^2+(y+4)^2=36$ centre $(6,-4)$ radius 6

b) $y=0 \Rightarrow (x-6)^2+16=36 \Rightarrow (x-6)^2=20$

$\Rightarrow x=6\pm\sqrt{20}=6\pm2\sqrt{5}$

so $AB=\underline{2\times2\sqrt{5}=4\sqrt{5}}$

4) a) $x=\frac{-10\pm\sqrt{100-4\times13}}{2}=-5\pm2\sqrt{3}$



b) $\underline{x<-5-2\sqrt{3}}$, $\underline{x>-5+2\sqrt{3}}$

SECTION 2

1) $AB^2=(16)^2+(30)^2=1156$ ✓ $AC^2=(8)^2+(2)^2=68$ ✓

(4) $BC^2=(8)^2+(32)^2=1088$ $1088+68=1156$ ✓

So ABC is ⊥ angled triangle & AB is diameter of circle ✓

(4) 2) $m=\frac{-1+3}{-2-4}=-\frac{1}{3}$ ✓ tangent: $m=3$ ✓ $\underline{y+1=3(x+2)}$ ✓ ✓

$(y=3x+5)$

3) a) AB⊥: $-\frac{1}{m}=\frac{6}{8} \Rightarrow m=-\frac{4}{3}$ ✓ midpoint $(6,-2)$ ✓

$y+2=-4/3(x-6)$ or $\underline{3y+4x-18=0}$ ✓

(10) AC⊥: $-\frac{1}{m}=\frac{1}{-7}$ $m=7$ ✓ midpoint $\left(\frac{11}{2},-\frac{11}{2}\right)$ ✓ ✓

$y+\frac{11}{2}=7(x-\frac{11}{2})$ or $\underline{2y-14x+88=0}$ ✓

b) $6y+8x-36=0$

(=) $6y-42x+264=0$

$50x=300$ ✓

$(x=6, y=-2)$ ✓ $(x-6)^2+(y+2)^2=25$ ✓

MIDPOINT AB

So SEMI-CIRCLE ⇒ ΔABC ⊥ △

4) a) $y=0 \Rightarrow x^2-4x+15=0$ $b^2-4ac=16-4\times15<0$ ✓

(5) So No solutions so does not intersect y Axis ✓

b) $P(8,1)$ $C(2,6)$ $CP=\sqrt{6^2+5^2}=\sqrt{61}$ ✓

(3) $(x-2)^2+(y-6)^2=25$ ✓ $r=5$ as $\sqrt{61}>5 \Rightarrow$ OUTSIDE CIRCLE ✓

c) $y=\frac{k}{3}+\frac{4}{3}x \Rightarrow (x-2)^2+\left(\frac{4}{3}x+\frac{k}{3}-6\right)^2=25$ ✓

$\Rightarrow x^2-4x+4+\frac{16}{9}x^2+\frac{8}{3}x\left(\frac{k}{3}-6\right)+\left(\frac{k}{3}-6\right)^2=25$

$\frac{25}{9}x^2+x\left(\frac{8}{9}k-20\right)+\left(\frac{k}{3}-6\right)^2-21=0$ ✓

$b^2-4ac=0$ if tangent (1 solution)

(7) $\left(\frac{8}{9}k-20\right)^2-4\left(\frac{25}{9}\right)\left(\left(\frac{k}{3}-6\right)^2-21\right)=0$ ✓

$\frac{64}{81}k^2-\frac{320}{9}k+400-\frac{100}{81}k^2+\frac{400}{9}k-\frac{500}{3}=0$ ✓

$\frac{36}{81}k^2-\frac{80}{9}k-\frac{700}{3}=0 \xrightarrow{\times 81} 36k^2-720k-18900=0 \xrightarrow{\div 36}$

$k^2-20k-525=0$ ✓ $k=35$ or -15 ✓

33 $(k-35)(k+15)$

SECTION 3

1) $(x-10)^2+(y+5)^2=10^2 \Rightarrow C(10,-5) \Rightarrow$ $CP: m=\frac{8}{6}=\frac{4}{3}$

$\Rightarrow PR$ is $y-3=-\frac{3}{4}(x-16)$

when $x=0$ $y=15$ $\Rightarrow \Delta PQR=\frac{1}{2}\times20\times16=80$

2) $AB^2=1^2+2^2=5$, $AC^2=3^2+4^2=25$ $BC^2=4^2+2^2=20$

So $AB^2+BC^2=AC^2 \Rightarrow$ AC is diameter $\left(x+\frac{1}{2}\right)^2+(y-3)^2=\frac{5}{2}$

3) a) $OQ^2=OT^2+TQ^2$

$\Rightarrow 11^2+k^2=36\times5+16 \Rightarrow k^2=75$ $k=5\sqrt{3}$

b) $(x-11)^2+(y-5\sqrt{3})^2=16$