PURE 10 SOLUTIONS (BINOMIAL EXPANSION & PROOF)

SECTION 1

1. $$\frac{1+\sqrt{10}}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3} = \underline{4\sqrt{10}+13}$$

2. $$3x = x\sqrt{5} + 2\sqrt{5} \qquad x(3-\sqrt{5}) = 2\sqrt{5}$$

$$x = \frac{2\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{10+6\sqrt{5}}{4} = \underline{\frac{5+3\sqrt{5}}{2}}$$

3. $$\frac{x^{3/2}-x}{x^{1/2}} = \underline{x - x^{1/2}}$$

4. $$\frac{x+1}{x^{1/2}+x^{-1/2}} = \frac{x^{1/2}(x+1)}{x^{1/2}(x^{1/2}+x^{-1/2})} = \frac{x^{1/2}(x+1)}{x+1} = \underline{x^{1/2}}$$

5. i) $x^2 - 20x + y^2 - 4y = 21$

$(x-10)^2 - 100 + (y-2)^2 - 4 = 21$

$(x-10)^2 + (y-2)^2 = 125$

$\therefore$ radius $= \sqrt{125}$ centre $(10, 2)$

ii) $x = 21$, $y = 0$

$(21-10)^2 + (0-2)^2 = 125$

$11^2 + 4 = 125$

$121 + 4 = 125$ ✓ verified.

$E \Rightarrow (0, ?)$

$x = 0$ $(0-10)^2 + (y-2)^2 = 125$

$100 + (y-2)^2 = 125$

$(y-2)^2 = 25$

$y - 2 = \pm 5$

$y = 7, -3$

$E \Rightarrow (0, 7)$

$D = (0, -3)$

$A \Rightarrow (?, 0)$ $y = 0$ $(x-10)^2 + (0-2)^2 = 125$

$(x-10)^2 + 4 = 125$

$(x-10)^2 = 121$

$x - 10 = \pm 11$

$x = -1, 21$

$A = (-1, 0)$

iii) Eq. of BE = ? Midpoint of BE $= \left(\frac{21+0}{2}, \frac{0+7}{2}\right) = \left(\frac{21}{2}, \frac{7}{2}\right)$

gradient $= \frac{0-7}{21-0} = \frac{-7}{21} = -\frac{1}{3}$

$\therefore$ perp. grad $= 3$. at C $x = 10$ $y = 2$

$y - \frac{7}{2} = 3\left(x - \frac{21}{2}\right)$ $\therefore$ $y = 3x - 28$ $2 = 3 \times 10 - 28$ ✓

Section 2

1a, $2(1+x)^{-3} = 2\left[1 + (-3)(x) + \frac{(-3)(-4)}{2}(x)^2 + \frac{(-3)(-4)(-5)}{6}x^3\right.$ M1

$= 2[1 - 3x + 6x^2 - 10x^3]$

$= 2 - 6x + 12x^2 - 20x^3$ A1

b, $(1-x)^{1/3} = 1 + (\tfrac{1}{3})(-x) + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})}{2}(-x)^2 + \frac{(\tfrac{1}{3})(-\tfrac{2}{3})(-\tfrac{5}{3})}{6}(-x)^3$ M1

$= 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{10}{162}x^3$ A1

2 a) $(4+x)^{1/2} = \left[4\left(1+\frac{x}{4}\right)\right]^{1/2} = 4^{1/2}\left(1+\frac{x}{4}\right)^{1/2}$ M1

$= 2\left(1+\frac{x}{4}\right)^{1/2}$

$= 2\left[1 + (\tfrac{1}{2})(\tfrac{x}{4}) + \frac{(\tfrac{1}{2})(-\tfrac{1}{2})}{2}\left(\frac{x}{4}\right)^2 + \frac{(\tfrac{1}{2})(-\tfrac{1}{2})(-\tfrac{3}{2})}{6}\left(\frac{x}{4}\right)^3\right]$

$= 2\left[1 + \frac{x}{8} - \frac{1}{128}x^2 + \frac{1}{1024}x^3 + \ldots\right]$ M1

$= 2 + \frac{x}{4} - \frac{1}{64}x^2 + \frac{1}{512}x^3 + \ldots$ A1 $|x| < 4$

2b) $(3-x)^{-3} = 3^{-3}\left(1-\frac{x}{3}\right)^{-3} = \frac{1}{27}\left(1-\frac{x}{3}\right)^{-3}$ M1

$= \frac{1}{27}\left[1 + (-3)(-\tfrac{x}{3}) + \frac{(-3)(-4)}{2}\left(-\frac{x}{3}\right)^2 + \frac{(-3)(-4)(-5)}{6}\left(\frac{-x}{3}\right)^3\right.$ M1

$= \frac{1}{27}\left[1 + x + \frac{2}{3}x^2 + \frac{10}{27}x^3 + \ldots\right]$

$= \frac{1}{27} + \frac{x}{27} + \frac{2}{81}x^2 + \frac{10}{729}x^3 + \ldots$ A1

$|x| < 3$

3 $(2+x)(4-2x)^{-1/2} = (2+x)(4^{-1/2})(1-\frac{x}{2})^{-1/2}$ M1

$$= \left(1 + (-\tfrac{1}{2})(-\tfrac{x}{2}) + \frac{(-\tfrac{1}{2})(-\tfrac{3}{2})}{2}(-\tfrac{x}{2})^2 + (-\tfrac{1}{2})(-\tfrac{3}{2}\ldots\right.$$

M1

$$= \left(1 + \frac{x}{4} + \frac{3}{32}x^2 + \ldots\right)$$

A1

$$(2+x) \times \frac{1}{2} \times \left(1 + \frac{x}{4} + \frac{3}{32}x^2\right)$$

$$2 \times \tfrac{1}{2} \times \tfrac{3}{32}x^2 + x \times \tfrac{1}{2} \times -\frac{x}{4} = \frac{3}{32}x^2 + \frac{x^2}{8} = \frac{7}{32}x^2$$

A1

4 $(1+ax)^b = 1 + b(ax) + \frac{b(b-1)}{2}(ax)^2$ M1

a) $\therefore \quad -6 = ab \quad , \quad \frac{b(b-1)a^2}{2} = 24$ M1

$a = \frac{-6}{b} \rightarrow , \quad \frac{b(b-1)}{2} \times \left(\frac{-6}{b}\right)^2 = 24$

$\cancel{b}(b-1)\frac{(36)}{b^{\cancel{2}}} = 48$

$(b-1)36 = 48b$

$36b - 36 = 48b$

$12b = -36$

$b = -3$ A1

$a = \frac{-6}{-3} = 2 \quad , a = 2$ A1

b) $x^3 \Rightarrow ? \quad (1+2x)^{-3}$

M1

$\Rightarrow 1 + \ldots + \ldots + \frac{(-3)(-4)(-5)}{6}(2x)^3$

$= -80x^3$

$\therefore \underline{-80}$ A1

5) a) Equilateral triangle

All angles $= 60°$ so none greater than $60°$

b) $a = -2$, $b = -1$

$-2 < -1$ but $(-2)^2 = 4$ and $(-1)^2 = 1$ so $a^2 > b^2$

c) $n = 4$

$4! + 1 = 25 = 5 \times 5$ so not prime

d) A vertical line $x = 2$

$x = 2$ cannot be written in the form $y = mx + c$ since y can take any value.

e) $k = 1/\pi$

$\frac{1}{\pi} \times \pi = 1 = 1/1 \Rightarrow$ rational

f) $a = 0$

$y = bx + c$ is a straight line

g) $n = 41$

$41^2 + 41 + 41 = 41(41 + 1 + 1) = 41 \times 43 \Rightarrow$ not prime

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(In each case, one mark for explanation on right lines, two marks for full explanation)

6 a) For a triangle, no side can be $\geq$ the sum of the other two sides.

Consider possible combinations of three numbers adding to 11

1	1	9	$9 > 1+1$ ✗
1	2	8	$8 > 1+2$ ✗
1	3	7	$7 > 1+3$ ✗
1	4	6	$6 > 1+4$ ✗
1	5	5	(✓)
2	2	7	$7 > 2+2$ ✗
2	3	6	$6 > 2+3$ ✗
2	4	5	(✓)
3	3	5	(✓)
3	4	4	(✓)

Sides of 1cm, 5cm, 5cm
2cm, 4cm, 5cm
3cm, 3cm, 5cm
3cm, 4cm, 4cm give triangles.

b) A number ending in 1 can be written as $10k+1$ etc.

Possible cases; $10k(10k+1) = 100k^2+10k = 10[10k^2+k] \Rightarrow$ ends in 0

$(10k+1)(10k+2) = 100k^2+30k+2 = 10[10k^2+3k]+2 \Rightarrow$ ends in 2

$(10k+2)(10k+3) = 100k^2+50k+6 = 10[10k^2+5k]+6 \Rightarrow$ ends in 6

$(10k+3)(10k+4) = 100k^2+70k+12 = 10[10k^2+7k+1]+2 \Rightarrow$ ends in 2

$(10k+4)(10k+5) = 100k^2+90k+20 = 10[10k^2+9k+2] \Rightarrow$ ends in 0

$(10k+5)(10k+6) = 100k^2+110k+30 = 10[10k^2+11k+3] \Rightarrow$ ends in 0

$(10k+6)(10k+7) = 100k^2+130k+42 = 10[10k^2+13k+4]+2 \Rightarrow$ ends in 2

$(10k+7)(10k+8) = 100k^2+150k+56 = 10[10k^2+15k+5]+6 \Rightarrow$ ends in 6

$(10k+8)(10k+9) = 100k^2+170k+72 = 10[10k^2+17k+7]+2 \Rightarrow$ ends in 2

$(10k+9)(10k+10) = 100k^2+190k+90 = 10[10k^2+19k+9] \Rightarrow$ ends in 0

All cases end in 0, 2 or 6

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7 3) a) $x^2+6x+11 = (x+3)^2+2$

$(x+3)^2 \geq 0$ for all real x

$\Rightarrow (x+3)^2+2 \geq 2$ for all real x

$\Rightarrow x^2+6x+11$ is positive for all real x

b) $b=a+1,\ c=a+2,\ d=a+3$

sum $a+b+c+d = a+a+1+a+2+a+3 = 4a+6$

$cd-ab = (a+2)(a+3)-a(a+1) = a^2+5a+6-a^2-a$

$= 4a+6$

$\Rightarrow a+b+c+d = cd-ab$

c) lengths of sides : $\sqrt{(5-2)^2+(2-1)^2} = \sqrt{10}$

$\sqrt{(4-2)^2+(5-1)^2} = \sqrt{20}$

$\sqrt{(4-5)^2+(5-2)^2} = \sqrt{10}$

Two sides equal $\Rightarrow$ isosceles

$(\sqrt{10})^2+(\sqrt{10})^2 = (\sqrt{20})^2$ $\Rightarrow$ satisfies Pythagoras. $\Rightarrow$ right angled.

d) Distinct real roots $\Rightarrow$ discriminant > 0

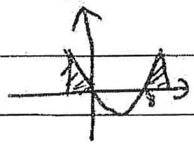
$a=1,\ b=k,\ c=2k$

$\Rightarrow$ discriminant $= k^2-4(2k) = k^2-8k$

$k^2-8k>0$

$\Rightarrow k(k-8)>0$

critical values $0, 8$



$k<0$ or $k>8$

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k is positive $\Rightarrow \underline{k>8}$

59 MARKS TOTAL