

Pure 1) - Vectors 1

Section 1

$$1a) \left(1 + \frac{x}{2}\right)^n = 1 + n\left(\frac{x}{2}\right) + \frac{n(n-1)}{2}\left(\frac{x}{2}\right)^2 + \frac{n(n-1)(n-2)}{6}\left(\frac{x}{2}\right)^3$$

$$\frac{n(n-1)}{2}\left(\frac{x}{2}\right)^2 = 30x^2$$

$$\frac{n^2 - n}{2} \times \frac{x^2}{4} = 30x^2$$

$$\frac{n^2 - n}{8} = 30$$

$$n^2 - n = 240$$

$$n^2 - n - 240 = 0$$

$$n = 16, \text{ ~~15~~}$$

$$b) 1 + 16\left(\frac{x}{2}\right) + \frac{16 \times 15}{2}\left(\frac{x}{2}\right)^2 + \frac{16 \times 15 \times 14}{6}\left(\frac{x}{2}\right)^3$$

$$1 + 8x + 120\left(\frac{x^2}{4}\right) + 560\left(\frac{x^3}{8}\right)$$

$$1 + 8x + 30x^2 + 70x^3$$

$$2a) f(2) = (2)^3 - 10(2) + 12 = 8 - 20 + 12 = 0$$

$$b) (x^3 - 10x + 12) \div (x - 2):$$

$$\begin{array}{r} x^2 \quad 2x \quad -6 \\ x \quad x^3 \quad 2x^2 \quad -6x \\ -2 \quad -2x^2 \quad -4x \quad 12 \end{array}$$

$$\underline{\underline{x = 2, 1 \pm \sqrt{7}}}$$

$$x^3 - 10x + 12 = (x^2 + 2x - 6)(x - 2)$$

$$\text{Solve: } x^2 + 2x - 6$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -6}}{2 \times 1} = -1 \pm \sqrt{7}$$

$$3a) \text{Midpoint: } \left(\frac{2+8}{2}, \frac{7+(-3)}{2}\right) = (5, 2)$$

$$\text{Length of diameter: } \sqrt{(8-2)^2 + (7-(-3))^2} = \sqrt{6^2 + 10^2} = \sqrt{136} = 2\sqrt{34}$$

$$\text{equation of circle: } (x-5)^2 + (y-2)^2 = 34$$

Section 2

1.

a) $\overline{BC} = \overline{AD}$
 $= q$ ✓

b) $\overline{DC} = \overline{AB}$
 $= p$ ✓

c) $\overline{BA} = -\overline{AB}$
 $= -p$ ✓

d) $\overline{CB} = \overline{DA}$
 $= -\overline{AD}$
 $= -q$ ✓

e) $\overline{AC} = \overline{AB} + \overline{BC}$
 $= p + q$ ✓

f) $\overline{BD} = \overline{BA} + \overline{AD}$
 $= q - p$ ✓

g) $\overline{DB} = \overline{DA} + \overline{AB}$
 $= -\overline{AD} + \overline{AB}$
 $= p - q$ ✓ 8

2. a)

$\overline{BD} = \overline{AE}$
 $= \overline{AB} + \overline{BE}$
 $= \overline{AB} + \overline{AF}$
 $= p + q$ ✓

b) $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CD}$ ✓
 $= 2p + q$ ✓

c) $\overline{CF} = \overline{CB} + \overline{BA} + \overline{AF}$
 $= -\overline{BC} - \overline{AB} + \overline{AF}$
 $= -2\overline{AB} + \overline{AF}$
 $= q - 2p$ ✓

d) $\overline{AG} = \frac{1}{2}\overline{AD}$
 $= \frac{1}{2}(2p + q)$ ✓ 9
 $= p + \frac{1}{2}q$ ✓

3. a)

Magnitude: $\sqrt{5^2 + 2^2} = \sqrt{29} = 5.39$ (to 3 sf) ✓
 Direction: $\cos^{-1}\left(\frac{5}{5.39\dots}\right) = 21.8^\circ$ (to 3 sf) ✓

b) Magnitude: $\sqrt{7^2 + 9^2} = \sqrt{130} = 11.4$ (to 3 sf) ✓
 Direction: $\cos^{-1}\left(\frac{7}{11.4\dots}\right) = 52.1^\circ$ (to 3 sf) ✓

c) Magnitude: $\sqrt{0^2 + (-5)^2} = \sqrt{25} = 5$ ✓
 Direction: $-\cos^{-1}\left(\frac{0}{5}\right) = -90^\circ$ (to 3 sf) ✓

d) Magnitude: $\sqrt{(-2)^2 + 3^2} = \sqrt{13} = 3.61$ (to 3 sf) ✓
 Direction: $\cos^{-1}\left(\frac{-2}{3.61\dots}\right) = 124^\circ$ (to 3 sf) ✓

e) Magnitude: $\sqrt{3^2 + (-5)^2} = \sqrt{34} = 5.83$ (to 3 sf) ✓
 Direction: $-\cos^{-1}\left(\frac{3}{5.83\dots}\right) = -59.0^\circ$ (to 3 sf) ✓

f) Magnitude: $\sqrt{(-6)^2 + (-5)^2} = \sqrt{61} = 7.81$ (to 3 sf) ✓
 Direction: $-\cos^{-1}\left(\frac{-6}{7.81\dots}\right) = -140^\circ$ (to 3 sf) ✓

g) Magnitude: $\sqrt{(-6)^2 + (-5)^2} = \sqrt{61} = 7.81$ (to 3 sf) ✓
 Direction: $-\cos^{-1}\left(\frac{-6}{7.81\dots}\right) = -140^\circ$ (to 3 sf) ✓

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4. a)
 x-component:
 $7 \cos 34^\circ = 5.80$ (to 3 sf) ✓
 y-component:
 $7 \sin 34^\circ = 3.91$ (to 3 sf) ✓
 So the vector is $5.80\mathbf{i} + 3.91\mathbf{j}$ ✓
- b)
 x-component:
 $9.6 \cos 78^\circ = 2.00$ (to 3 sf) ✓
 y-component:
 $9.6 \sin 78^\circ = 9.39$ (to 3 sf) ✓
 So the vector is $2.00\mathbf{i} + 9.39\mathbf{j}$ ✓
- c)
 x-component:
 $12 \sin 48^\circ = 8.92$ (to 3 sf) ✓
 y-component:
 $12 \cos 48^\circ = 8.03$ (to 3 sf) ✓
 So the vector is $8.92\mathbf{i} + 8.03\mathbf{j}$ ✓
- d)
 x-component:
 $4 \sin 0^\circ = 0$ ✓
 y-component:
 $4 \cos 0^\circ = 4$ ✓
 So the vector is $4\mathbf{j}$ ✓
- e)
 x-component:
 $8.3 \cos 64^\circ = 3.64$ (to 3 sf) ✓
 y-component:
 $8.3 \sin 64^\circ = 7.46$ (to 3 sf) ✓
 So the vector is $-3.64\mathbf{i} + 7.46\mathbf{j}$ ✓
- f)
 x-component:
 $6.2 \cos(180+55)^\circ = -3.56$ (to 3 sf) ✓
 y-component:
 $6.2 \sin(180+55)^\circ = -5.08$ (to 3 sf) ✓
 So the vector is $-3.56\mathbf{i} - 5.08\mathbf{j}$ ✓

5. a)
 $\mathbf{p} + \mathbf{q} = (2\mathbf{i} - \mathbf{j}) + (-2\mathbf{i} + 3\mathbf{j})$ ✓
 $= 2\mathbf{j}$ ✓
- b)

$$\mathbf{p} - \mathbf{r} = (2\mathbf{i} - \mathbf{j}) - (4\mathbf{i} + \mathbf{j})$$

$$= -2\mathbf{i} - 2\mathbf{j}$$

c)
 $2\mathbf{q} - \mathbf{p} = 2(-2\mathbf{i} + 3\mathbf{j}) - (2\mathbf{i} - \mathbf{j})$ ✓
 $= -6\mathbf{i} + 7\mathbf{j}$ ✓

d)
 $2\mathbf{p} + 3\mathbf{r} = 2(2\mathbf{i} - \mathbf{j}) + 3(4\mathbf{i} + \mathbf{j})$ ✓
 $= 16\mathbf{i} + \mathbf{j}$ ✓

e)
 $|\mathbf{p}| = |2\mathbf{i} - \mathbf{j}|$
 $= \sqrt{2^2 + (-1)^2}$ ✓
 $= \sqrt{5}$ ✓
 $= 2.24$

f)
 $|\mathbf{q} + \mathbf{r}| = |(-2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} + \mathbf{j})|$ ✓
 $= |2\mathbf{i} + 4\mathbf{j}|$
 $= \sqrt{2^2 + 4^2}$ ✓
 $= \sqrt{20}$ ✓
 $= 4.47$

(13)

6. a)
 $u = -10, v = -1$ ✓ ✓

b)
 If $\mathbf{p} = k\mathbf{r}$, then from the x-components
 $k = 0.75 \Rightarrow u = -4.5$ ✓

(4)

7. a)
 $|\mathbf{p}| = 5$, so required vector is
 $4\mathbf{p} = -12\mathbf{i} + 16\mathbf{j}$ ✓ ✓

b)
 $\hat{\mathbf{p}} = \frac{1}{5}\mathbf{p}$ ✓
 $= -0.6\mathbf{i} + 0.8\mathbf{j}$ ✓

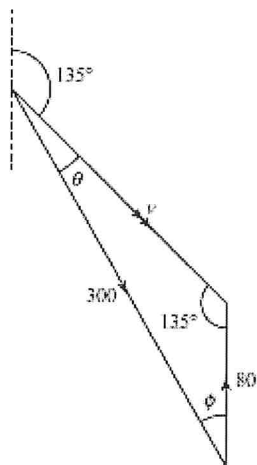
(4)

(18)

TOTAL: (70)

Section 3

1.



a In the diagram:

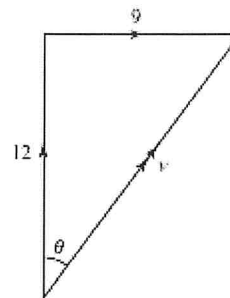
$$\theta = \sin^{-1}\left(\frac{80 \sin 135^\circ}{300}\right) = 10.9^\circ$$

The bearing is 145.9°

b $\phi = 180^\circ - 135^\circ - 10.9^\circ = 34.1^\circ$

$$v = \frac{300 \sin 34.1^\circ}{\sin 135^\circ} = 238 \text{ km h}^{-1}$$

2. a)



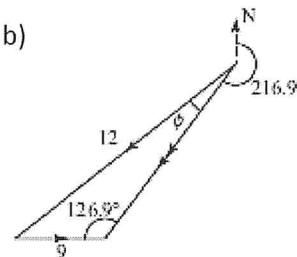
From the diagram:

$$\theta = \tan^{-1}\left(\frac{9}{12}\right) = 36.87^\circ$$

$$v = \sqrt{12^2 + 9^2} = 15$$

The velocity is 15 km hr^{-1} on a bearing of 036.9°

b)



In the diagram:

$$\phi = \sin^{-1}\left(\frac{9 \sin 126.9^\circ}{12}\right) = 36.87^\circ$$

Steer on a bearing of $180^\circ + \theta + \phi = 253.7^\circ$