Pure ~~10~~ 12 - Vectors 2

Section 1

1a) i) $\overline{AB} = \overline{AE} + \overline{EB}$
$= \overline{AE} + 2\overline{AE}$
$= 3\overline{AE}$
$= 3\underline{p}$

ii) $\overline{BC} = \overline{BA} + \overline{AC}$
$= -3\underline{p} + 2\overline{AF}$
$= -3\underline{p} + 2\underline{q}$

iii) $\overline{DF} = \overline{DC} + \overline{CF}$
$= -\overline{BC} + -\overline{AF}$
$= 3\underline{p} - 2\underline{q} - \underline{q}$
$= 3(\underline{p} - \underline{q})$

b) $\overrightarrow{EF} = -p + q \quad \overrightarrow{FD} = -3p + 3q = 3\overrightarrow{EF}$ $\therefore$ $\overrightarrow{EF}$ and $\overrightarrow{FD}$ are parallel and since they have point F in common, they are collinear.

2a) $(1-2x)^9 = 1 + 9(-2x) + \frac{9(9-1)}{2}(-2x)^2 + \frac{9(9-1)(9-2)}{6}(-2x)^3$

$= 1 - 18x + 36 \times 4x^2 + 84 \times -8x^3$

$= 1 - 18x + 144x^2 - 672x^3$

b) $1 - 2x = 0.98$
$-2x = -0.02$
$x = 0.01$

$1 - 18(0.01) + 144(0.01)^2 - 672(0.01)^3$

$= 1 - 0.18 + 0.0144 - 0.000672$

$= 0.8337$

3a) $4 \times 0 + 3x = 12$
$x = 4$
$(4, 0)$

$4y + 3 \times 0 = 12$
$y = 3$
$(0, 3)$

b) reflection in x axis:
$(4,0)$, $(0,-3)$
$\frac{y-0}{0-(-3)} = \frac{x-4}{4-0}$ $\quad 3x - 4y - 12 = 0$
$\frac{y}{3} = \frac{x-4}{4}$

reflection in y axis: $(-4,0), (0,3)$
$\frac{y-0}{0-3} = \frac{x-(-4)}{-4-0}$

$\frac{y}{-3} = \frac{x+4}{-4}$ $\quad 3x - 4y + 12 = 0$

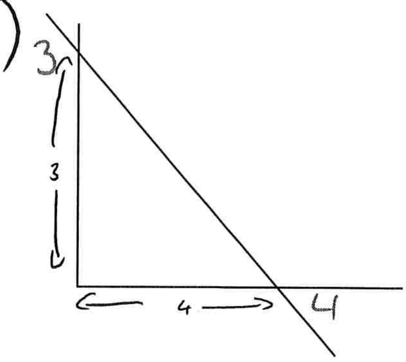
$-4y = -3x - 12$

reflection in $y=x$: $(0,4), (3,0)$
$\frac{y-4}{4-0} = \frac{x-0}{0-3}$

$\frac{y-4}{4} = \frac{x}{-3}$ $\quad 4x + 3y - 12 = 0$

$-3y + 12 = 4x$

c)



Area $= \frac{1}{2} b \times h$

$= \frac{1}{2} \times 4 \times 3$

$= 6$

2

Section 2

1.

Express the displacement between successive checkpoints as vectors:

$$\begin{pmatrix}7\\6\end{pmatrix}-\begin{pmatrix}1\\11\end{pmatrix}=\begin{pmatrix}6\\-5\end{pmatrix}$$

$$\begin{pmatrix}13\\1\end{pmatrix}-\begin{pmatrix}7\\6\end{pmatrix}=\begin{pmatrix}6\\-5\end{pmatrix}$$

These vectors are equal and share a common point.

So the checkpoints are collinear and can be reached by travelling in a straight line.

4

2.

Final position of A is A', with position:

$$\mathbf{a}' = (2\mathbf{i}+5\mathbf{j})+(2\mathbf{i}-3\mathbf{j})$$
$$= (4\mathbf{i}+2\mathbf{j})\text{ m}$$

Displacement of B is:

$$-3(2\mathbf{i}-3\mathbf{j}) = (-6\mathbf{i}+9\mathbf{j})\text{ m}$$

Final position of B is B', with position:

$$\mathbf{b}' = (6\mathbf{i}+3\mathbf{j})+(-6\mathbf{i}+9\mathbf{j})$$
$$= 12\mathbf{j}\text{ m}$$

$$\overrightarrow{A'B'} = \mathbf{b}'-\mathbf{a}'$$
$$= -4\mathbf{i}+10\mathbf{j}$$

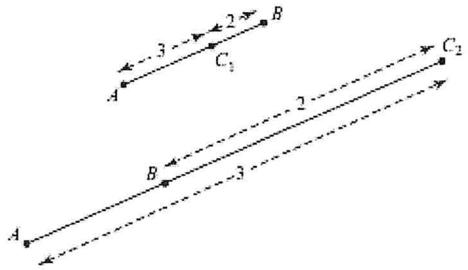
Distance $= A'B'$

$$= \sqrt{(-4)^2+10^2}$$
$$= 10.8\text{ m}$$

(or $2\sqrt{29}$)

6

3.



The diagram shows the two possible situations.

C_1 lies between A and B

$$\mathbf{c}_1 = \mathbf{a}+\frac{3}{5}(\mathbf{b}-\mathbf{a})$$
$$= 7.8\mathbf{i}+4\mathbf{j}$$

C_2 lies on AB produced, with $AC_2 = 3AB$

$$\mathbf{c}_2 = \mathbf{a}+3(\mathbf{b}-\mathbf{a})$$
$$= 27\mathbf{i}+16\mathbf{j}$$

5

4.

Express the position of towns B, C and D as vectors (from A).

$$\mathrm{B} = \begin{pmatrix}-200\\0\end{pmatrix}$$

$$\mathrm{C} = \begin{pmatrix}0\\100\end{pmatrix}$$

$$\mathrm{D} = \begin{pmatrix}\cos 45^\circ \times 283\\ \sin 45^\circ \times 283\end{pmatrix}$$
$$= \begin{pmatrix}200\\200\end{pmatrix}$$

Calculate the displacement vectors $\overrightarrow{BC}$ and $\overrightarrow{CD}$

$$\overrightarrow{BC} = \begin{pmatrix}0\\100\end{pmatrix}-\begin{pmatrix}-200\\0\end{pmatrix}$$
$$= \begin{pmatrix}200\\100\end{pmatrix}$$

$$\overrightarrow{CD} = \begin{pmatrix}200\\200\end{pmatrix}-\begin{pmatrix}0\\100\end{pmatrix}$$
$$= \begin{pmatrix}200\\100\end{pmatrix}$$

$\overrightarrow{BC} = k\overrightarrow{CD}$ (where $k=1$)

So the positions of the shops are collinear

6

3

7 After t s the first particle is at $2t\mathbf{i}+(4-t)\mathbf{j}$

The second particle is at $(6-t)\mathbf{i}+(8-3t)\mathbf{j}$

When $t=2$, both particles are at $(4\mathbf{i}+2\mathbf{j})$ m

3

5 a)

The displacement of A is

$\mathbf{p}=(7\mathbf{i}+4\mathbf{j})-(3\mathbf{i}+\mathbf{j})$

$=(4\mathbf{i}+3\mathbf{j})$ m

$|\mathbf{p}|=\sqrt{4^2+3^2}=5$

So the unit vector $\hat{\mathbf{p}}=\frac{\mathbf{p}}{5}$

$=0.8\mathbf{i}+0.6\mathbf{j}$

Position of B is $((2+0.8d)\mathbf{i}+0.6d\mathbf{j})$ m

b)

If $d=15$, position of B is $(14\mathbf{i}+9\mathbf{j})$ m

So $\overrightarrow{AB}=7\mathbf{i}+5\mathbf{j}$

The distance AB is $|AB|=\sqrt{7^2+5^2}$

$=8.60$ m

8

6 The resultant displacement is:

$$\overrightarrow{PQ}=\begin{pmatrix}6\cos 10^\circ\\6\sin 10^\circ\end{pmatrix}+\begin{pmatrix}7\cos 70^\circ\\7\sin 70^\circ\end{pmatrix}+\begin{pmatrix}-5\cos 25^\circ\\5\sin 25^\circ\end{pmatrix}$$

$$=\begin{pmatrix}3.77\\9.73\end{pmatrix}$$

Length of tunnel is $PQ=\sqrt{3.77^2+9.73^2}$

$=10.4$ km

The journey would reduce by:

$(6+7+5)-10.4=7.6$ km

The tunnel should not be built.

7

TOTAL: 39