PURE 14 DIFFERENTIATION

SECTION 1

1. i) $\underline{6i + 3j}$ ii) $\underline{-6i - 5j}$

2 i) $\sqrt{3^2+4^2} = \underline{5}$

ii) $\sqrt{3^2+6^2} = \sqrt{45} = \underline{3\sqrt{5}}$

iii) $\sqrt{1^2+1^2} = \underline{\sqrt{2}}$

3. $\underline{-i + 8j}$

4. $\underline{a} = 3\underline{i} - 4\underline{j}$ $\underline{b} = 2\underline{i} + 5\underline{j}$ $\underline{c} = -\underline{i} - 3\underline{j}$

i) $\underline{b} + 2\underline{a} = 2\underline{i} + 5\underline{j} + 2(3\underline{i} - 4\underline{j})$

$= \underline{8i - 3j}$

ii) $2\underline{c} - \underline{b} = 2(-\underline{i} - 3\underline{j}) - (2\underline{i} + 5\underline{j})$

$= \underline{-4i - 11j}$

iii) $\underline{a} - \underline{b} + 3\underline{c} = (3\underline{i} - 4\underline{j}) - (2\underline{i} + 5\underline{j}) + 3(-\underline{i} - 3\underline{j})$

$= \underline{-2i - 18j}$

5. i) $(x+1)^6 = 1^6 + \binom{6}{1}1^5x^1 + \binom{6}{2}1^4x^2$

$+ \binom{6}{3}1^3x^3 + \binom{6}{4}1^2x^4 + \binom{6}{5}1x^5 + x^6$

$= \underline{\underline{1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6}}$

ii) $(x-2)^5 = (-2+x)^5$

$= (-2)^5 + \binom{5}{1}(-2)^4x^1 + \binom{5}{2}(-2)^3x^2$

$+ \binom{5}{3}(-2)^2x^3 + \binom{5}{4}(-2)x^4 + x^5$

$= \underline{\underline{-32 + 80x - 80x^2 + 40x^3 - 10x^4 + x^5}}$

iii) $(2x+1)^4 = (1+2x)^4$

$= 1^4 + \binom{4}{1}1^3(2x)^1 + \binom{4}{2}1^2(2x)^2 + \binom{4}{3}1^1(2x)^3 + (2x)^4$

$= \underline{\underline{1 + 8x + 24x^2 + 32x^3 + 16x^4}}$

iv) $(2-3x)^3 = 2^3 + \binom{3}{1}2^2(-3x)^1 + \binom{3}{2}2^1(-3x)^2 + (-3x)^3$

$= \underline{\underline{8 - 36x + 54x^2 - 27x^3}}$

6 i) $\left(1+\frac{x}{2}\right)^9 = 1^9 + \binom{9}{1}1^8\left(\frac{x}{2}\right) + \binom{9}{2}1^7\left(\frac{x}{2}\right)^2 + \binom{9}{3}1^6\left(\frac{x}{2}\right)^3 + \ldots$

$= 1 + \frac{9x}{2} + \frac{9x^2}{4} + \frac{21}{2}x^3 + \ldots$

ii) $x = 0.1 \quad \therefore \quad 1 + \frac{x}{2} = 1.05$

$\therefore \quad 1.05^9 \simeq 1 + \frac{9}{2} \times 0.1 + \frac{9 \times 0.1^2}{4} + \frac{21}{2} \times 0.1^3$

$= 1.\overset{55}{\cancel{5}}05$

iii) % error $= \dfrac{1.05^9 - 1.\overset{55}{\cancel{5}}05}{1.05^9} \times 100\%$

$= \cancel{5.85\%} \quad -0.053\%$

SECTION 2

1. i) 2 ↘

ii) $3x^2 - 5$ ↘

iii) $-\frac{3}{x^4}$ ↘

iv) $\frac{1}{3}x^{-2/3}$ ↘

v) $-\frac{2}{x^2} + \frac{6}{x^3}$ ↘↘

vi) $2x^{-1/2} + \frac{3}{2}x^{-3/2}$ ↘↘

vii) $-15x^{-6} + 14x^{-8}$ ↘↘

viii) $\frac{4}{3}x^{-1/3} + \frac{10}{3}x^{-5/3}$ ↘↘

ix) $12x^3 - 6x^{\frac{1}{2}} - \frac{1}{x^2}$ x) $2x^2 + 2$

xi) $\frac{5}{2}x^{3/2} - x^{-1/2}$ xii) $\frac{1}{x^2} - \frac{3}{x^3}$

2. $y = 12x - x^3$

i) $\frac{dy}{dx} = 12 - 3x^2$ $\therefore$ at $x = 0$ $\frac{dy}{dx} = 12$.

ii) $\frac{dy}{dx} = 0$ $\therefore$ $3x^2 - 12 = 0$ $\therefore$ $x = \pm 2$

$x = 2$ $y = 12 \times 2 - 2^3 = 16$

$x = -2$ $y = 12 \times -2 - (-2)^3 = -24 + 8 = -16$

(2, 16) and (−2, −16)

3. i) $y = 2x - \frac{1}{x}$ $\therefore$ $\frac{dy}{dx} = 2 + \frac{1}{x^2}$

$x = 1$ $\therefore$ $\frac{dy}{dx} = 2 + 1 = 3$.

ii) $y = 3 - \sqrt{x}$ $\therefore$ $\frac{dy}{dx} = -\frac{1}{2}x^{-1/2}$

$x = 4$ $\frac{dy}{dx} = -\frac{1}{4}$.

iii) $y = x^2\sqrt{x} = x^{5/2}$

$\therefore \frac{dy}{dx} = \frac{5}{2}x^{3/2}$

at $x = 1$ $\frac{dy}{dx} = \frac{5}{2}$

4. $y = x^3 + 2x^2$ $\therefore \frac{dy}{dx} = 3x^2 + 4x$

$\frac{dy}{dx} = 4$ $\therefore 3x^2 + 4x = 4$

$\therefore 3x^2 + 4x - 4 = 0$

$\therefore (3x - 2)(x + 2) = 0$

$\therefore x = 2/3, -2.$

5. $y = ax^3 + bx$ $\therefore \frac{dy}{dx} = 3ax^2 + b$

$x = 1 \Rightarrow y = 8, \frac{dy}{dx} = 12$

$\therefore a + b = 8$
$3a + b = 12$

$\therefore 2a = 4 \therefore a = 2 \therefore b = 6$

6. $y = 5x^2$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5(x+h)^2 - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{5x^2 + 5h^2 + 10xh - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(5h + 10x)}{h}$$

$$= \lim_{h \to 0} 5h + 10x$$

$$= \underline{\underline{10x}}$$

SECTION 3

1. $y = (2x+3)^2$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(2(x+h)+3)^2 - (2x+3)^2}{h}$$

$$= \lim_{h \to 0} \frac{(2x+3+2h)^2 - (2x+3)^2}{h}$$

$$= \lim_{h \to 0} \frac{(2x+3)^2 + 4h^2 + 4h(2x+3) - (2x+3)^2}{h}$$

$$= \lim_{h \to 0} \quad 4h + 4(2x + 3)$$

$$= \underline{\underline{8x + 12}}$$

2. $y = (2x+3)^3$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(2(x+h)+3)^3 - (2x+3)^3}{h}$$

$$= \lim_{h \to 0} \frac{(2x+3+2h)^3 - (2x+3)^3}{h}$$

$$= \lim_{h \to 0} \frac{4h^2(2x+3) + 8h^3 + 8h^2(2x+3) + 6h(2x+3)^2}{h}$$

$$= \underline{\underline{6(2x+3)^2}}$$

3. $f(x) = (ax+b)^n$

$$\therefore \underline{\underline{f'(x) = na(ax+b)^{n-1}}}$$