

# Pure 22 Logarithms ANSWERS

## Section 1

a)  $A(0, 5) B(0, e^2)$  so  $AB = e^2 - 5$

b)  $e^{x+2} = 3 + 2e^x$   $e^{x+2} - 2e^x = 3$

$$e^x(e^2 - 2) = 3$$

$$e^x = \frac{3}{e^2 - 2}$$

$$\Rightarrow x = \ln\left(\frac{3}{e^2 - 2}\right)$$

∴ from  $y = e^x \times e^2$   
 $= \frac{3e^2}{e^2 - 2}$

## Section 2

1) a)  $\log_2(21)$  b)  $\log_2(9)$  c)  $\log_5(80)$  d)  $\log_6\left(\frac{64}{81}\right)$

⑧ e)  $\log_{10}\left(5 \times 6 \div \frac{1}{4}\right) = \log_{10}(120)$

2) a)  $\log_2(8) = 3$  b)  $\log_6(36) = 2$  c)  $\log_{12}(9 \times 16) = 2$

⑫ d)  $\log_8\left(\frac{250}{125}\right) = \log_8(2) = \frac{1}{3}$  e)  $\log_{10}\left(\frac{400}{40}\right) = 1$

3) a)  $3\log x + 4\log y + \log z$  b)  $5\log x - 2\log y$  c)  $2\log a + 2\log x$

⑩ d)  $\log x + \frac{1}{2}\log y - \log z$  e)  $\frac{1}{2}\log a + \frac{1}{2}\log x$

4) a)  $6 \cdot 23$  b)  $66 \cdot 7$  c)  $126$  d)  $1.66$  e)  $-3 \cdot 22$  f)  $1.31$

g)  $x - 1 = 2x + 2$  h)  $-\frac{0.34}{2}$  i)  $9 - 3x = x$

⑫  $x = -3$

$x = \frac{9}{4}$

j)  $4 - 3x = 2x + 10$

$5x = -6$

$x = -\frac{6}{5}$

5d)  $(2^x - 5)(2^x - 1) = 0$   $x = \log_2 5 = 2.32$

(3)  $x = 0$

b)  $(3^x - 4)(3^x - 11) = 0$   $x = \log_3 4 = 1.26$

$x = \log_3 11 = 2.18$

(3)

c)  $7^{2x} - 7^{x+1} + 12 = 0$   $\rightarrow (7^x - 4)(7^x - 3) = 0$

$7^{2x} - 7(7^x) + 12 = 0$   $x = \log_7 4 = 0.712$

$x = \log_7 3 = 0.565$

(3)

d)  $\log_{10} x^2 - \log_{10} (2x-1) = 1$

$\log_{10} \left( \frac{x^2}{2x-1} \right) = 1 \Rightarrow \frac{x^2}{2x-1} = 10$

$x^2 - 20x + 10 = 0$   $x = \frac{20 \pm \sqrt{20^2 - 4 \times 1 \times 10}}{2}$

$= 19.5$  or  $0.513$

(3)

e)  $3(3^{2x}) - 26(3^x) - 9 = 0$

$(3(3^x) + 1)(3^x - 9) = 0$

~~$x = \log_3 \left( \frac{1}{3} \right)$  NO ANSWER~~

$x = 2$

(3)

f)  $\log_e (x^2 - 1) = 1.2$

$x^2 - 1 = e^{1.2} = 3.32 \dots$

$x^2 = 4.32 \dots$

(3)

$x = \pm 2.08$

② 6) a)  $x = 3^2 = 9$       b)  $\log_3 \left( \frac{y^2}{y+4} \right) = 2$

④  $y^2 = 9y + 36$   
 $y^2 - 9y - 36 = 0$   
 $(y - 12)(y + 3) = 0$        $\boxed{y = 12}$   
 $\boxed{y = -3}$

④ c)  $\log_3 z = \frac{1}{\log_2 3} = 4 \log_2 3 \Rightarrow (\log_2 3)^2 = \frac{1}{4} \log_2 3 = -\frac{1}{2}$   
 $z = 9$  OR  $z = \frac{1}{9}$

7)  $\log_3 (2 - 3x) = \log_3 (6x^2 - 19x + 2) / \log_3 9 = \log_3 (6x^2 - 19x + 2)$

⑤  $\Rightarrow (2 - 3x)^2 = (6x^2 - 19x + 2)$   
 $0 = +3x^2 + 7x + 2 = (3x + 1)(x + 2)$   
 $x = -\frac{1}{3}, x = -2$

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### Section 3

1) a)  $3 = \log_2 y - \log_2 x^2 = \log_2 \left( \frac{y}{x^2} \right) \Rightarrow \frac{y}{x^2} = 2^3 = 8$

so  $y = 8x^2$  (=  $14x - 3$  in part b)

b)  $0 = 8x^2 - 14x + 3 \Rightarrow (4x - 1)(2x - 3) = 0$

$x = \frac{1}{4}$  OR  $\frac{3}{2}$

d)  $\log_2 \left( \frac{3}{2} \right) = 0.585$

c)  $\log_2 \left( \frac{1}{4} \right) = -2$

because  $2^{-2} = \frac{1}{4}$

2)  $2y = \ln(x+3) - 1$

$\Rightarrow e^{\ln(x+3) - 1} - x + 2 = 0 \Rightarrow (x+3) \times \frac{1}{e} - x + 2 = 0$

$\Rightarrow x \left( \frac{1}{e} - 1 \right) = -2 - \frac{3}{e} \Rightarrow x = \frac{2 + \frac{3}{e}}{1 - \frac{1}{e}} = 4.91$

$y = 0.53$

