

PURE 26 SOLUTIONS

(63)

1. $u_1=2$ $u_2=7$ $u_3=17$ $u_4=37$ ✓ ✓ ✓ ✓ (2)

2. (a) 2, 7, 27, 107 ✓ divergent ✓
 (b) 3.2, 3.04, 3.008, 3.0016 ✓ convergent ✓ (4)

3. (a) $\sum_{r=1}^4 (2r+3) = 5+7+9+11$ ✓

(b) $\sum_{r=0}^3 2 \times 3^r = 2+6+18+54$ ✓

-1 if no +5

(c) $\sum_{r=1}^4 (-1)^r r^2 = -1+4-9+16$ ✓ (3)

4. (a) $u_1 = 45 - 6 = 39$ ✓
 $u_2 = 45 - 12 = 33$ ✓

(b) $d = -6$ ✓

(c) $S_n = 0 \Rightarrow \frac{n}{2} \{2 \times 39 + (n-1)(-6)\} = 0$ ✓

$\times 2 \rightarrow n(78 - 6n + 6) = 0$

$n(84 - 6n) = 0$ ✓

$n=0$ or $n = \frac{84}{6} = 14$

$\rightarrow n = 14$ ✓ (5)

5. (a) 13, 9, 5, ... (2)

(b) -108 (3)

(c) $\sum_{n=1}^{20} (17 - 4n)$ (3)

(d) use $S_n = \frac{n}{2} \{2a + (n-1)d\} = \frac{n}{2} \{2(13) + (n-1)(-4)\}$

$S_n = \frac{n}{2} \{26 - 4n + 4\} = \frac{n}{2} \{30 - 4n\} = n(15 - 2n)$ ✓ ✓ (3)

$$6. (a) 2(7) - 1 + 2(8) - 1 + 2(9) - 1 + 2(10) - 1 = 64 \quad (2)$$

$$(b) (1+3) + (1+6) + (1+9) + \dots + (1+60) \quad (2)$$

$$4 + 7 + 10 + \dots + 61 = 650$$

$$7. (a) 3, 7, 11, 15 \quad \checkmark \quad (d) 5, 2, 5, 2 \quad \checkmark$$

$$(b) 2, 7, 22, 67 \quad \checkmark \quad (e) -1, 14, -46, 194 \quad \checkmark \quad (6)$$

$$(c) -2, 1, 7, 19 \quad \checkmark \quad (f) 10, 3, 2.3, 2.23 \quad \checkmark$$

8. Possible answers are:

$$(a) u_{n+1} = u_n + 4, u_1 = 5 \quad \checkmark$$

$$(b) u_{n+1} = 3u_n, u_1 = 1 \quad \checkmark$$

$$(c) u_{n+1} = u_n - 18, u_1 = 62 \quad \checkmark \quad (6)$$

$$(d) u_{n+1} = \frac{1}{2}u_n, u_1 = 120 \quad \checkmark$$

$$(e) u_{n+1} = 2u_n + 1, u_1 = 4 \quad \checkmark$$

$$(f) u_{n+1} = 4u_n - 1, u_1 = 1 \quad \checkmark$$

$$9. (a) u_2 = \frac{1}{2}(R+6) \quad \checkmark$$

$$u_3 = \frac{1}{2}\left[R + 3 \times \frac{1}{2}(R+6)\right] = \frac{1}{4}(5R+18) \quad \checkmark \quad (2)$$

$$(b) \frac{1}{4}(5R+18) = 7 \rightarrow R = 2 \quad \checkmark \quad u_4 = \frac{1}{2}(2+21) = \frac{23}{2} \quad \checkmark \quad (2)$$

$$10. (a) AP: a=2 \quad S_{80} = \frac{80}{2}(2+160) = 6480 \quad \checkmark$$

$$L=160 \quad (2)$$

$$n=80$$

$$(b) AP: a=3 \quad S_{66} = \frac{66}{2}(3+198) = 6633 \quad \checkmark$$

$$L=198$$

$$n=66$$

$$\begin{aligned}
 10. (c) \text{ AP: } a &= 30 & a + (n-1)d &= L \\
 L &= 300 & 30 + 6(n-1) &= 300 \\
 d &= 6 & 30 + 6n - 6 &= 300 \\
 & & 6n &= 276 \\
 & & n &= 46 \quad \checkmark
 \end{aligned}$$

$$\Rightarrow S_{46} = \frac{46}{2}(30+300) = 7590 \quad \checkmark \quad (2)$$

$$\begin{aligned}
 11. (a) 2t - (5-t) &= (6t-3) - 2t \\
 3t - 5 &= 4t - 3 \quad \checkmark \Rightarrow t = -2 \quad \checkmark \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 (b) u_5 &= 7 \quad u_6 = -4 \Rightarrow d = -11 \\
 a + 4(-11) &= 7 \Rightarrow a = 51 \quad \checkmark
 \end{aligned}$$

$$S_{18} = \frac{18}{2}[102 + 17(-11)] = -765 \quad \checkmark \quad (2)$$

$$\begin{aligned}
 12. (a) \frac{k+4}{k-8} &= \frac{3k+4}{k+4} \Rightarrow (k+4)^2 = (3k+2)(k-8) \\
 & \quad \checkmark \quad k^2 - 15k - 16 = 0 \\
 & \quad (k+1)(k-16) = 0 \\
 & \quad k > 0 \Rightarrow k = 16 \quad \checkmark \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 (b) u_1 &= 8 \Rightarrow a = 8 \\
 u_2 &= 20 \quad r = \frac{5}{2} \quad \checkmark \quad u_6 = 8\left(\frac{5}{2}\right)^5 = 781.25 \quad \checkmark \quad (2)
 \end{aligned}$$

$$(c) S_{10} = 50857.3 \quad \text{use } S_n = \frac{a(r^n - 1)}{r - 1} \quad \checkmark \quad (1)$$

$$\begin{aligned}
 13. (a) \text{ After } 4^{\text{th}} \text{ bounce, reaches:} \\
 3(0.6)^4 &= 0.3888 \text{ m} \quad \checkmark \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 (b) \text{ total distance: } &= \\
 & h + 2[0.6h + h(0.6)^2] + h(0.6)^3 + \dots \\
 &= h + 2S \text{ of GP } \checkmark \quad a = 0.6h \quad \checkmark \quad (3) \\
 & \quad r = 0.6 \quad \checkmark \\
 &= h + \frac{(2+0.6h)}{(1-0.6)} = h + 3h = 4h \text{ meters.} \quad \checkmark
 \end{aligned}$$